Learning at Scale: Deep, Distributed and Multi-dimensional

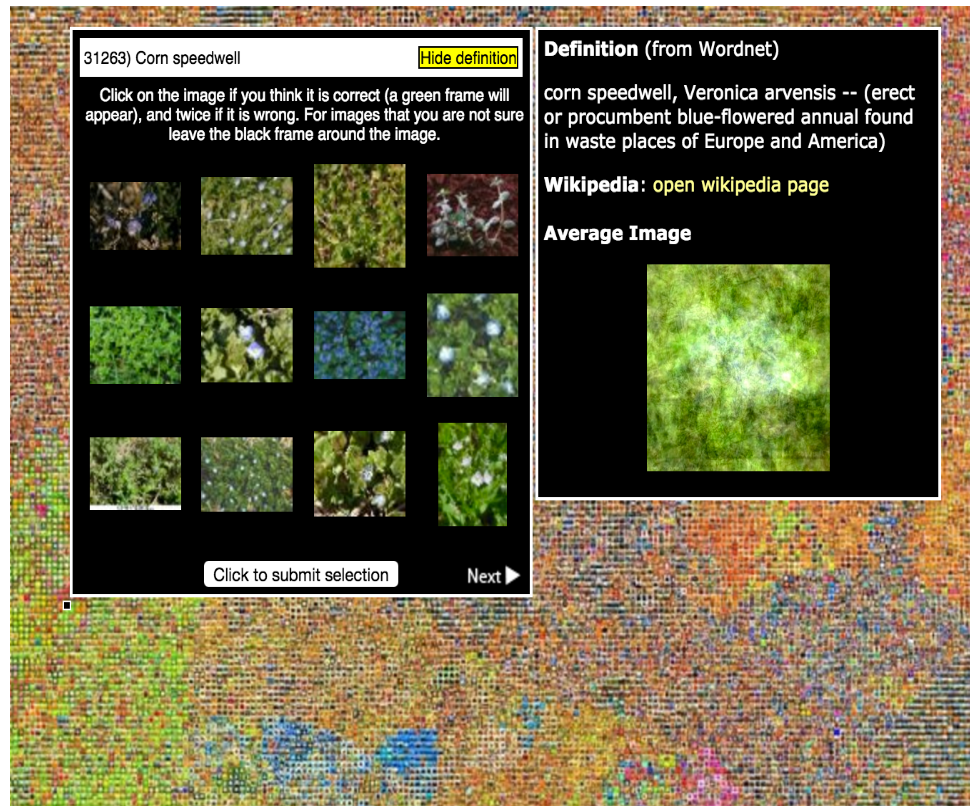
Anima Anandkumar

Amazon AI & Caltech

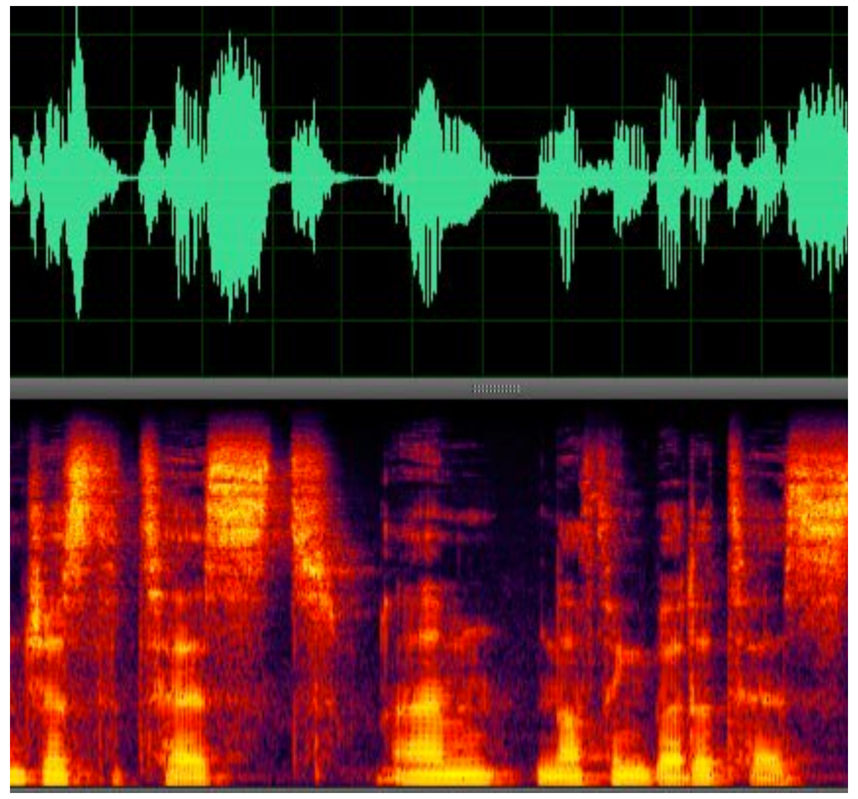
Deep Learning

Significantly improve many applications on multiple domains

image understanding



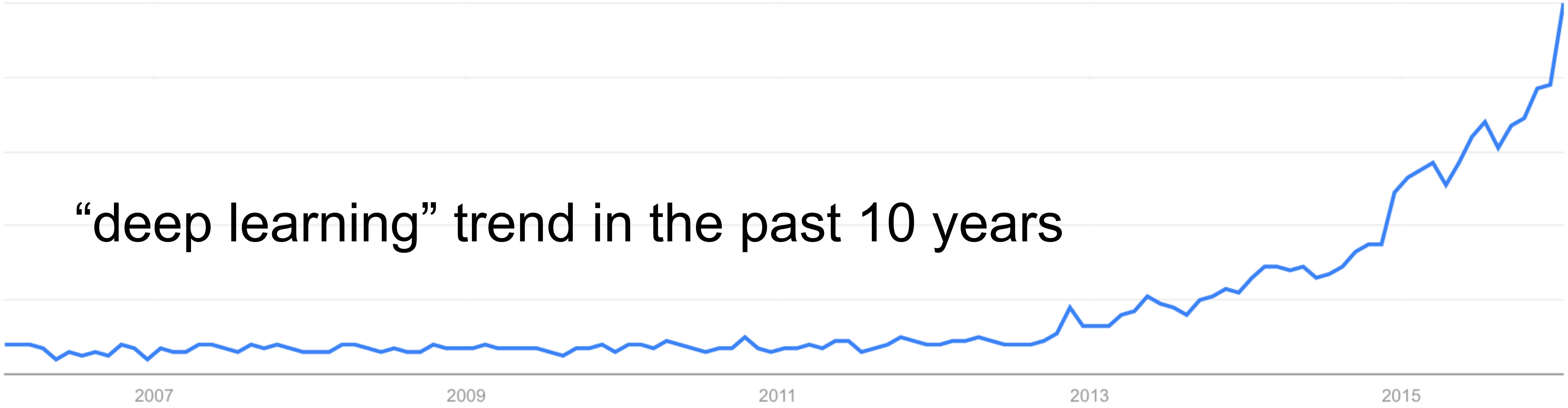
speech recognition



natural language processing

autonomy

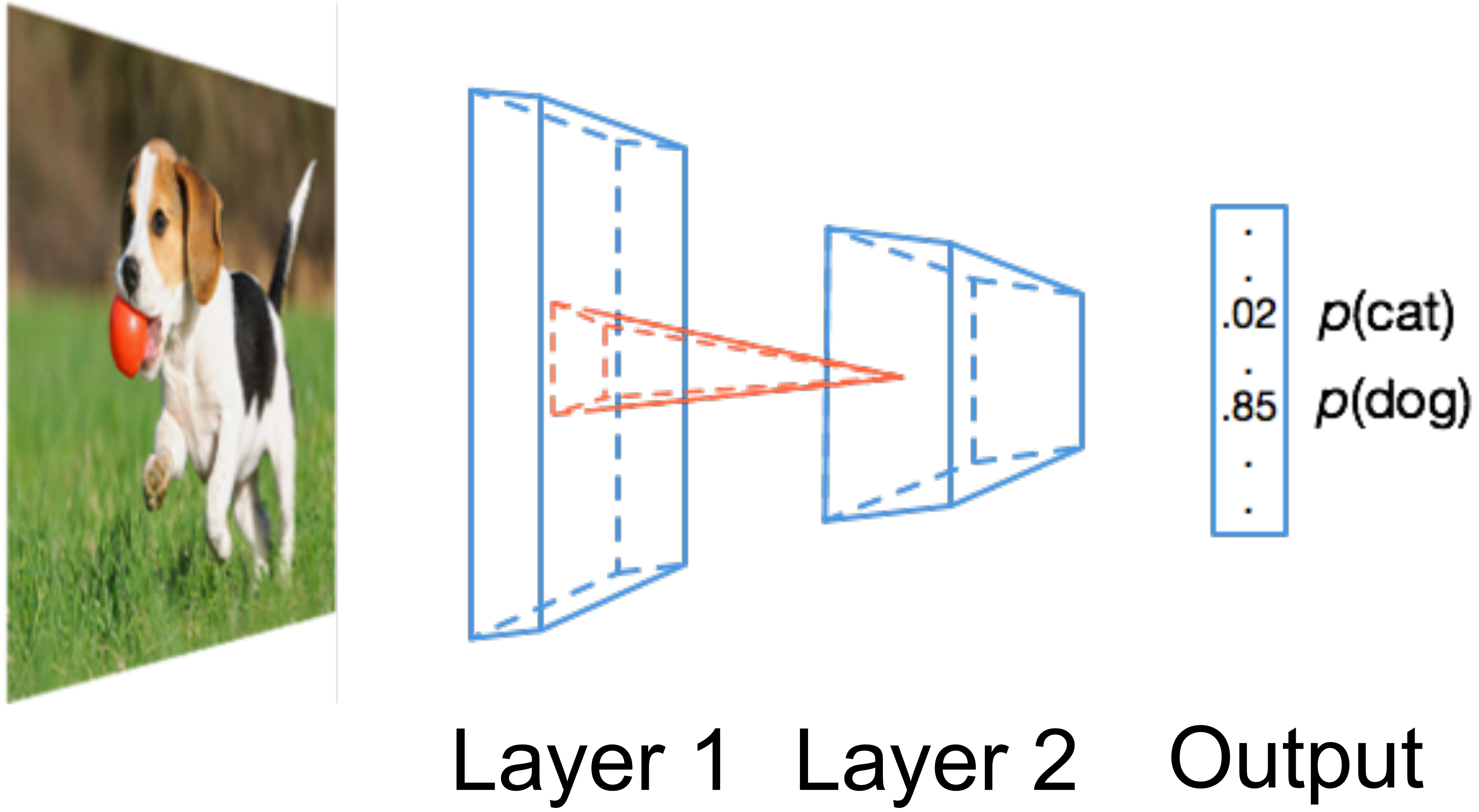
…



“deep learning” trend in the past 10 years

Image Classification

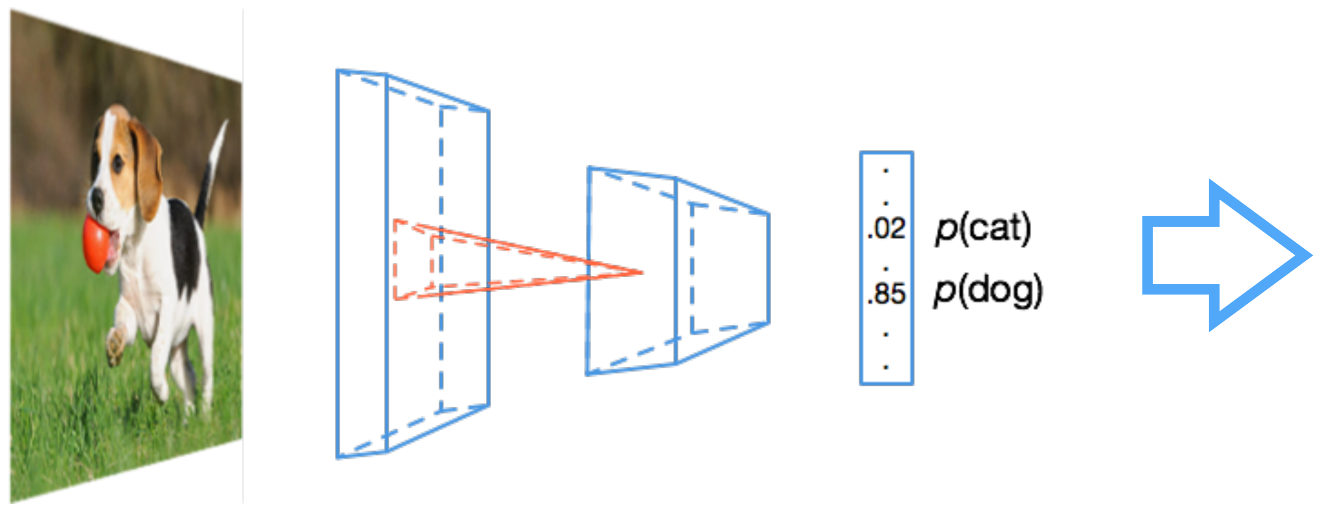
multilevel feature extractions from raw pixels
to semantic meanings



explore spatial information with convolution layers

Image Classification

State-of-the-art networks have tens to hundreds layers



- Hard to define the network
  - the definition of the inception network has >1k lines of codes in Caffe
- A single image requires billions floating-point operations
  -  Intel i7 ~500 GFLOPS
  -  Nvidia Titan X: ~5 TFLOPS
- Memory consumption is linear with number of layers

Outline

3. MXNet

- Imperative and Declarative Programming
- Language Support
- Backend and Automatic Parallelization

image credit - wikipedia

amazon web services

Caffe

ResNet-101-deploy.prototxt

```
layer {
        bottom: "data"
        top: "conv1"
        name: "conv1"
        type: "Convolution"
        convolution_param {
                num_output: 64
                kernel_size: 7
                pad: 3
                stride: 2
```

....

(4K lines of codes)

- Protobuf as the interface
- Portable
  - caffe binary + protobuf model
- Reading and writing protobuf are not straightforward

Tensorflow

Implement Adam

```
# m_t = beta1 * m + (1 - beta1) * g_t
m = self.get_slot(var, "m")
m_scaled_g_values = grad.values * (1 - beta1_t)
m_t = state_ops.assign(m, m * beta1_t,
                       use_locking=self._use_locking)
m_t = state_ops.scatter_add(m_t, grad.indices, m_scaled_g_values,
                            use_locking=self._use_locking)
```

> 300 lines of codes

- A rich set of operators (~2000)
- The codes are not very easy to read, e.g. not python-like

Keras

```
model = Sequential()
model.add(Dense(512, activation='relu',
                input_shape=(784,)))
model.add(Dropout(0.2))
model.add(Dense(512, activation='relu'))
model.add(Dropout(0.2))
model.add(Dense(10, activation='softmax'))

model.compile(...)
model.fit(...)
```

- Simple and easy to use
- Difficult to implement sophisticated algorithms

Pytorch & Chainer

```
class Net(nn.Module):
    def __init__(self, input_size, hidden_size, num_classes):
        super(Net, self).__init__()
        self.fc1 = nn.Linear(input_size, hidden_size)
        self.relu = nn.ReLU()
        self.fc2 = nn.Linear(hidden_size, num_classes)

    def forward(self, x):
        out = self.fc1(x)
        out = self.relu(out)
        out = self.fc2(out)
        return out
```

- Flexible
- Complicate programs might be slow to run

MXNet

Implement Resnet

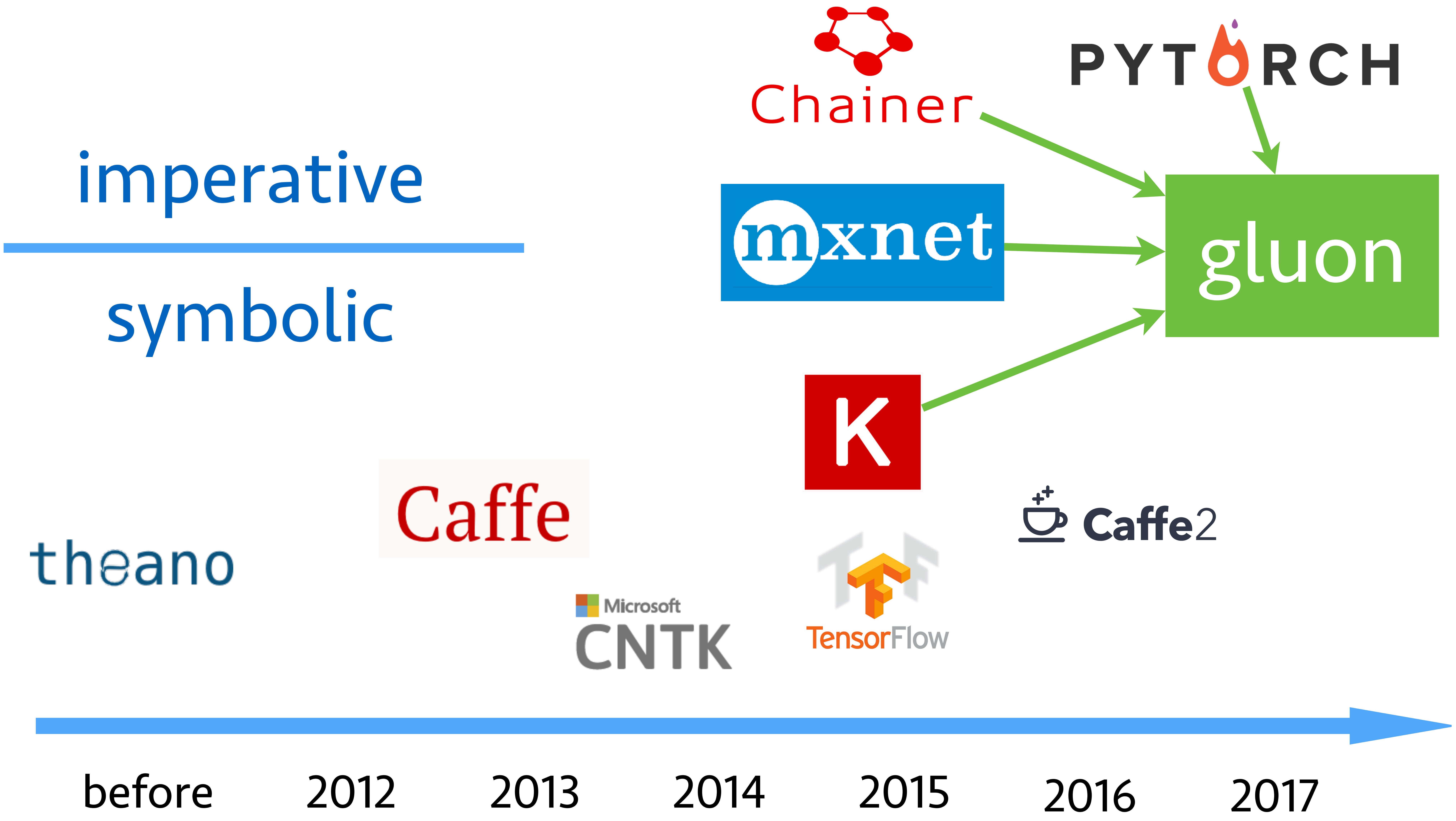
```
bn1 = sym.BatchNorm(data=data, fix_gamma=Fal
act1 = sym.Activation(data=bn1, act_type='re
conv1 = sym.Convolution(data=act1, num_filte
```

Implement Adam

```
coef2 = 1. - self.beta2**t
lr *= math.sqrt(coef2)/coef1

weight -= lr*mean/(sqrt(variance) + self.epsilon)
```

- Symbolic on network definition
- Imperative on tensor computation
- Huh.., not good enough


imperative
symbolic
Chainer
PYTORCH
mxnet
gluon
K
Caffe
Caffe2
theano
Microsoft
CNTK
TensorFlow
before
2012
2013
2014
2015
2016
2017

Gluon at a glance

```
net = gluon.nn.Sequential()
with net.name_scope():
    net.add(gluon.nn.Dense(128, activation='relu'))
    net.add(gluon.nn.Dense(64, activation='relu'))
    net.add(gluon.nn.Dense(10))

loss = gluon.loss.SoftmaxCrossEntropyLoss()

for data, label in get_batch():
    with autograd.record():
        l = loss(net(data), label)
    l.backward()
    trainer.step(batch_size=data.shape[0])
```

`net.hybridize()` converts from imperative to symbolic execution

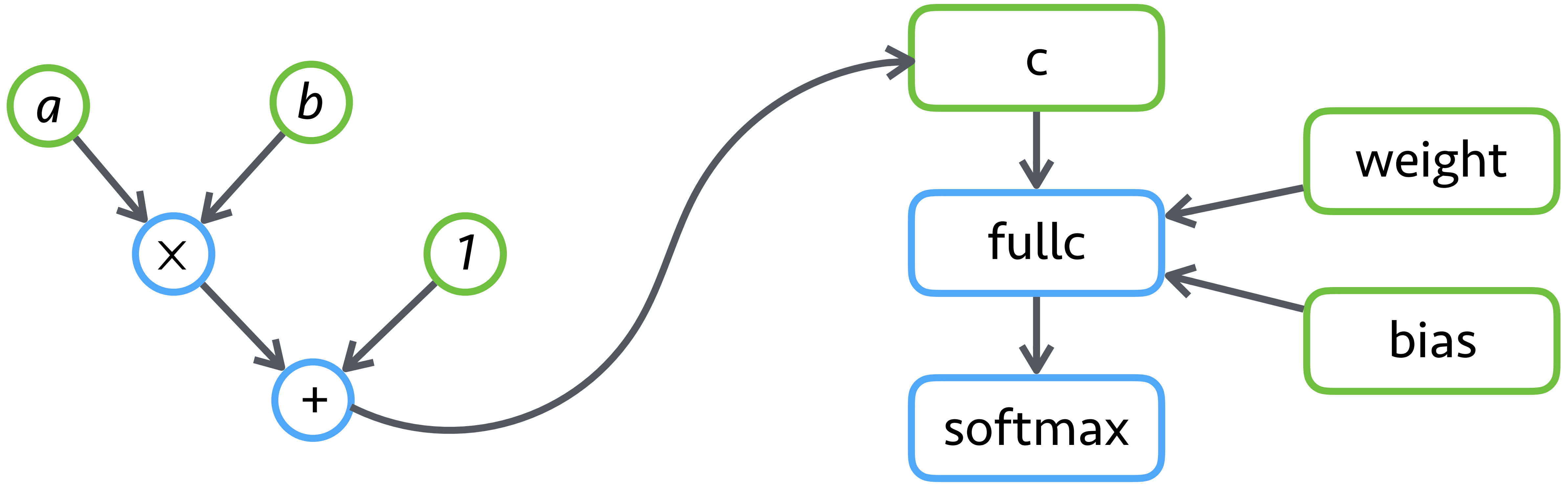
Back-end System

```python
import mxnet as mx
a = mx.nd.zeros((100, 50))
b = mx.nd.ones((100, 50))
c = a * b
c += 1
```

```python
import mxnet as mx
net = mx.symbol.Variable('data')
net = mx.symbol.FullyConnected(
        data=net, num_hidden=128)
net = mx.symbol.SoftmaxOutput(data=net)
texec = mx.module.Module(net)
texec.forward(data=c)
texec.backward()
```

Front-end

Back-end

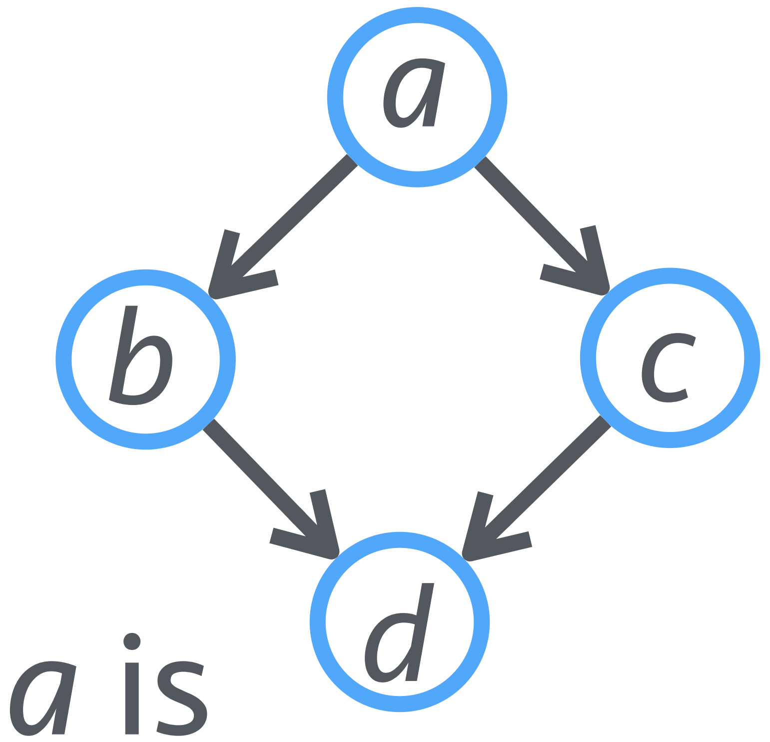


- ✧ Optimization
  - ✓ Memory optimization
  - ✓ Operator fusion
- ✧ Scheduling
  - ✓ Auto-parallelization

Memory Optimization

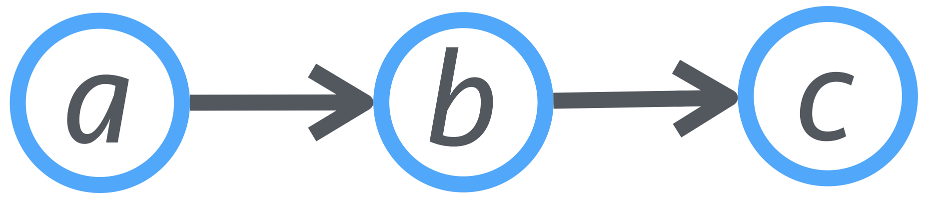
Traverse the computation graph to reduce the memory footprint with time complexity linear in graph size

aliveness analysis



now *a* is deletable

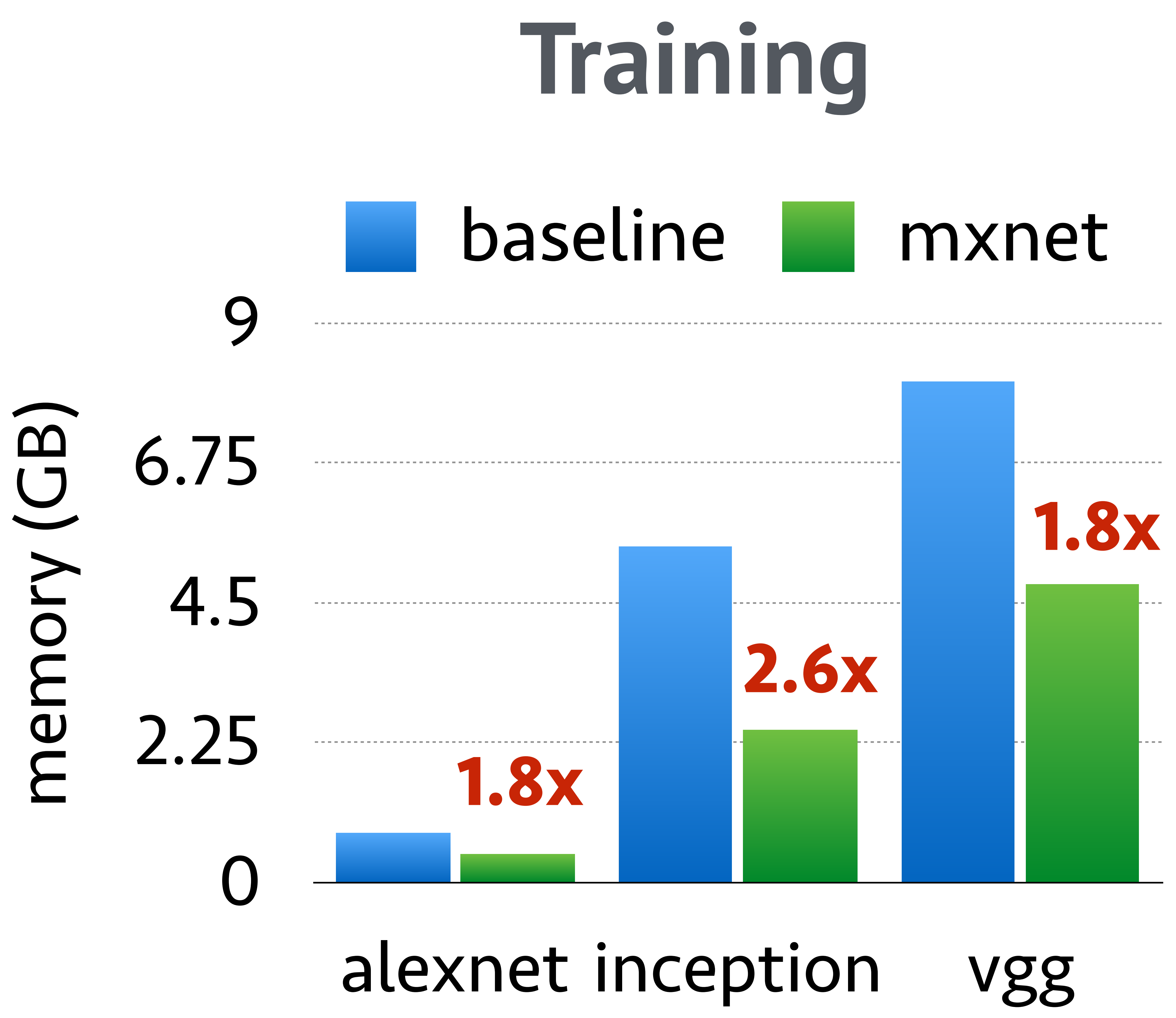
shared space between variables

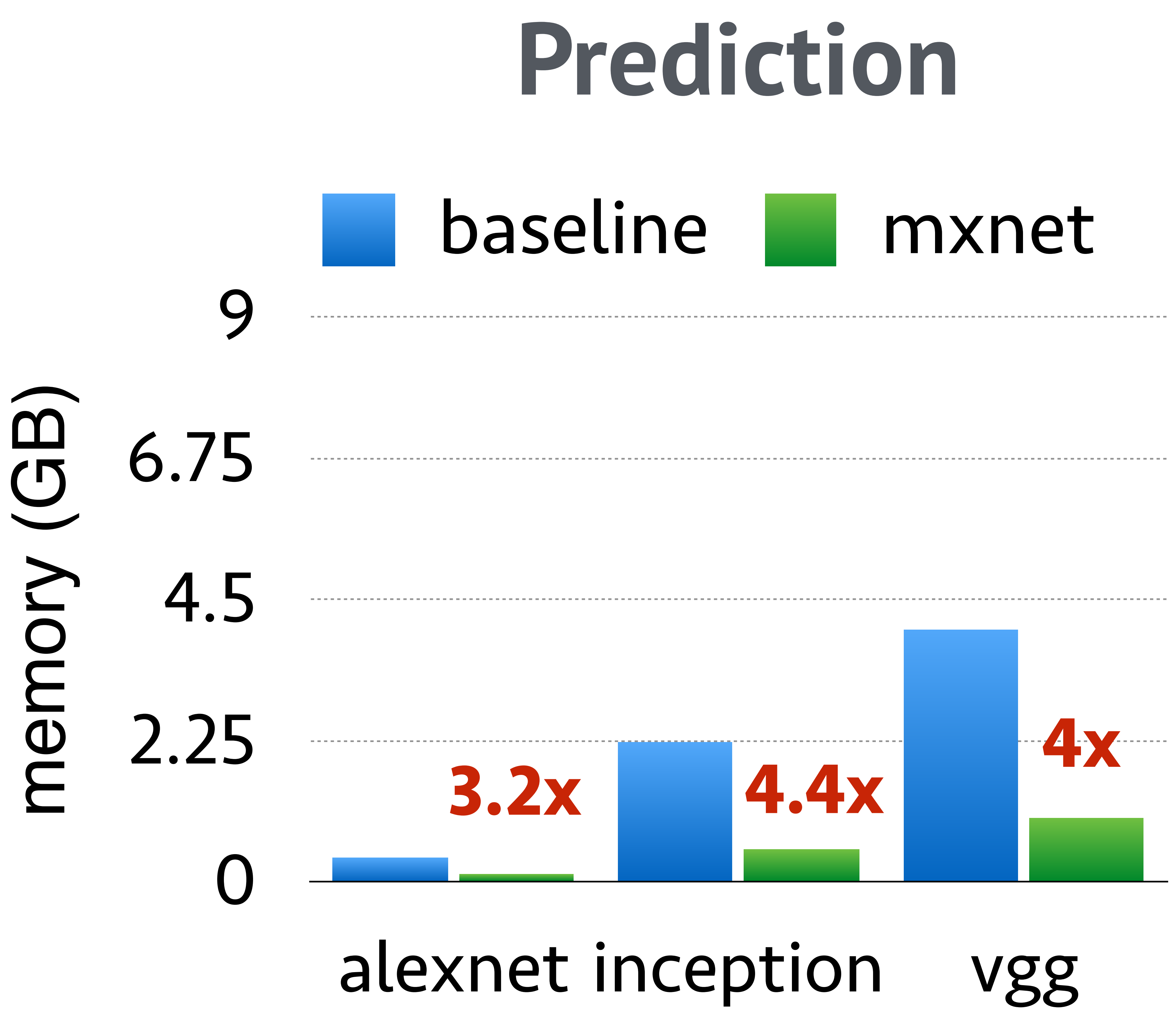


share *a* and *b*

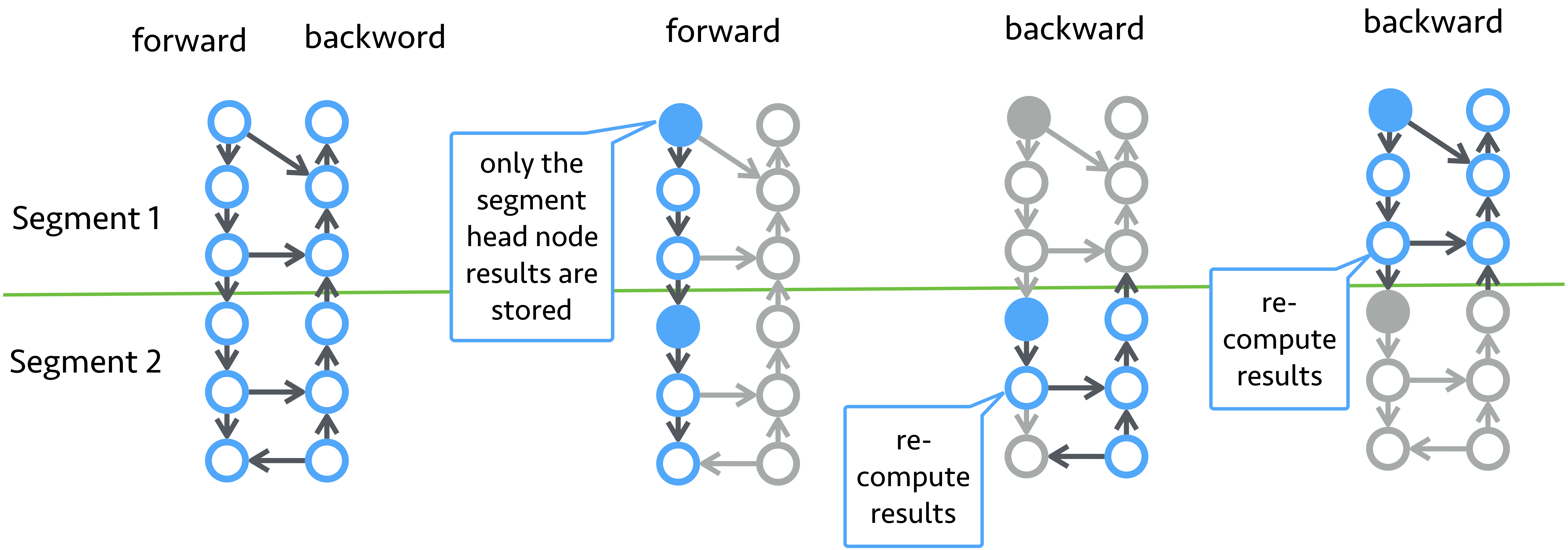
Results for Deep CNNs

IMAGENET winner neural networks





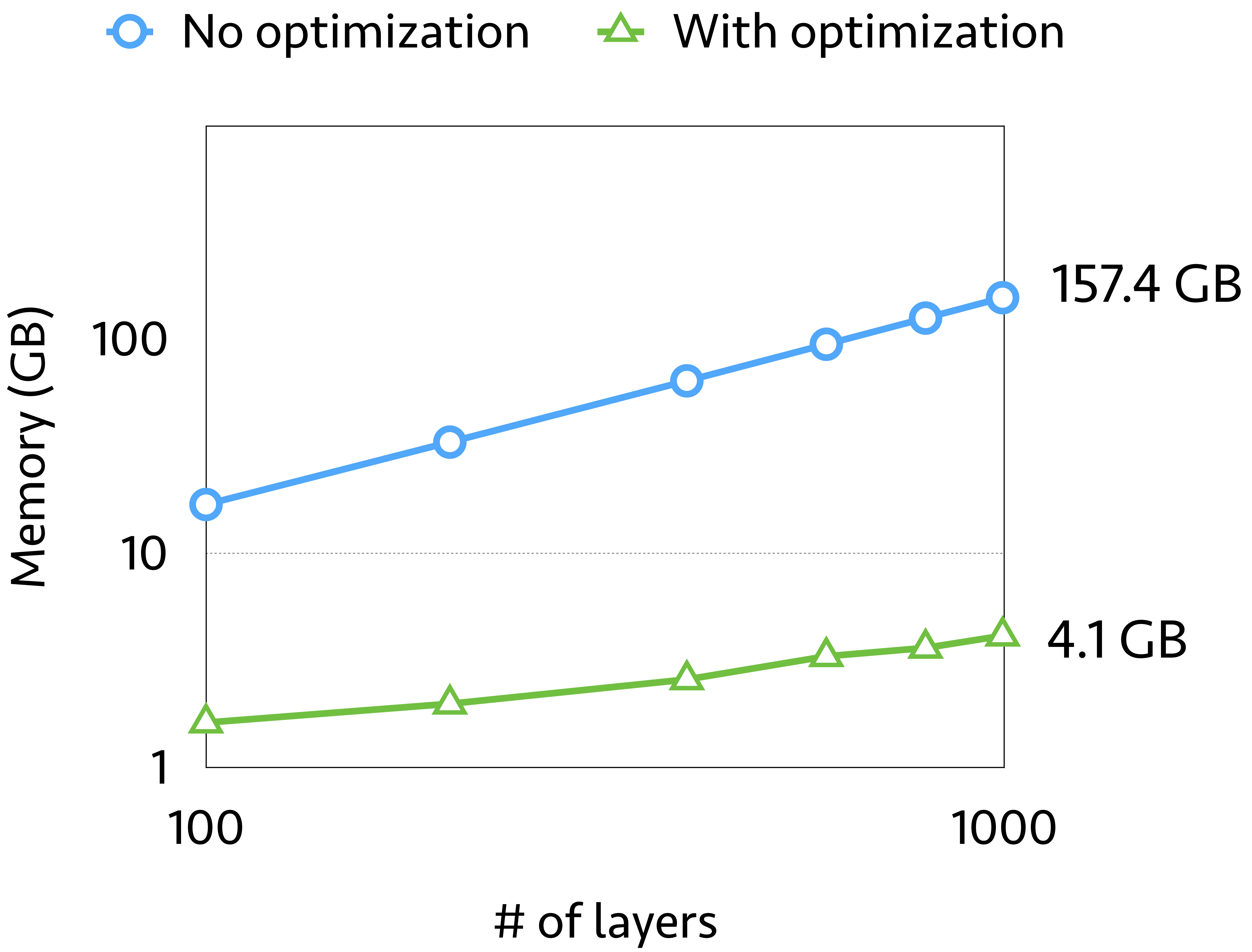
Trade Computation for Memory



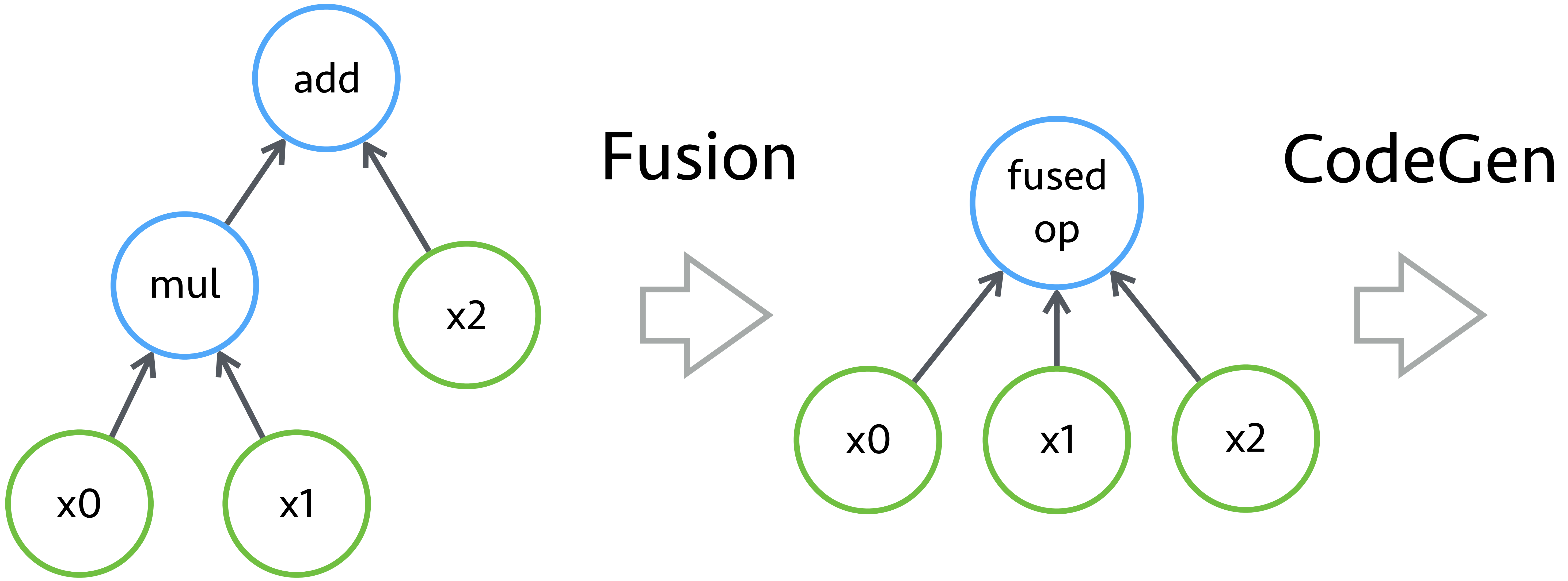
- Needs an extra forward pass
- Reduces the memory complexity from $O(n)$ to $O(\sqrt{n})$, where *n* is the number of layers

Results on ResNet

- Batch size = 32
- Increase 30% computation cost when optimization is applied



Operator Fusion and Runtime Compilation




```
extern "C" __global__ fusion_kernel (uint32_t num_element,
    float *x0, float *x1, float *x2, float *y) {
  int global_idx = blockIdx.x * blockDim.x + threadIdx.x;
  if (global_idx < num_element)
    y[global_idx] = (x0[global_idx] * x1[global_idx]) + x2[global_
}
```

Fuse Adam into a single operator

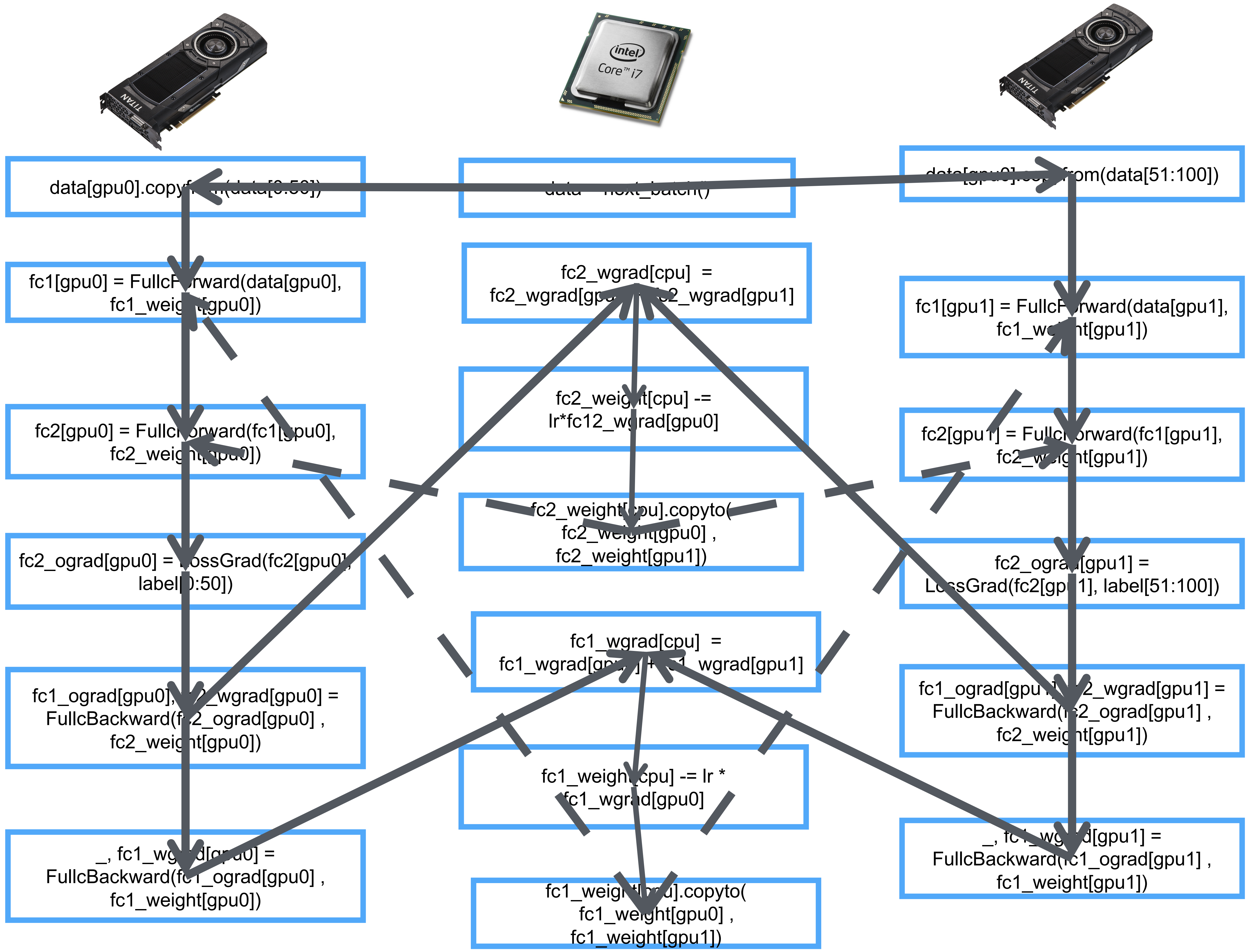
```
variance *= self.beta2
variance += (1 - self.beta2) * square(grad, out=grad)

coef1 = 1. - self.beta1**t
coef2 = 1. - self.beta2**t
lr *= math.sqrt(coef2)/coef1
```

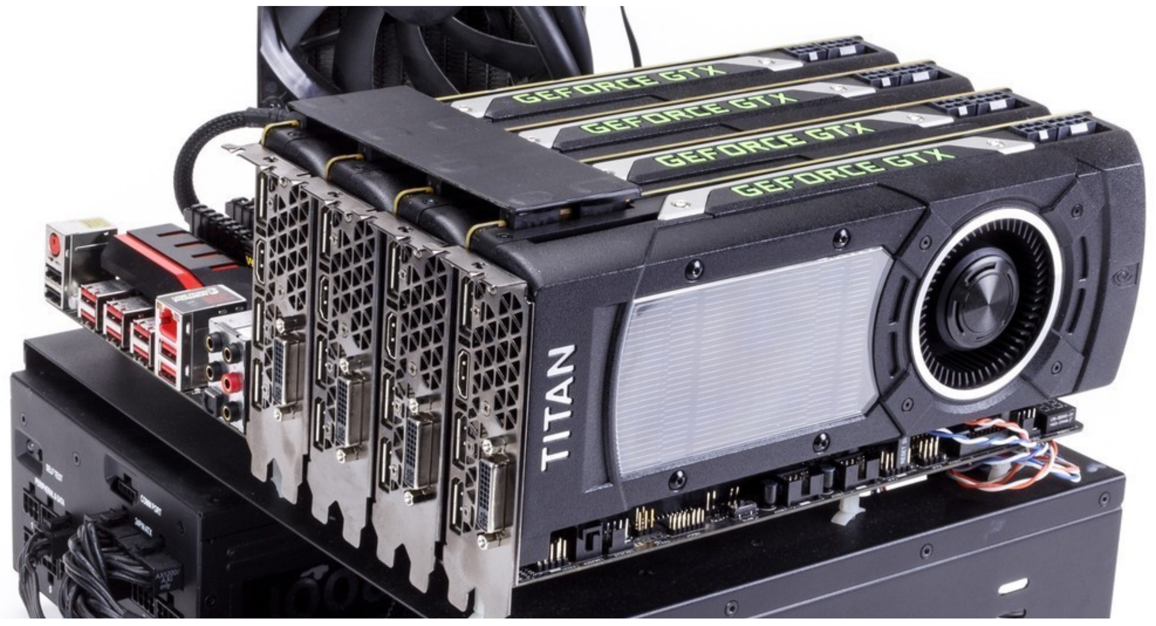
20% performance improvement on ResNet

Writing Parallel Programs is Painful

Dependency graph for 2-layer neural networks with 2 GPUs



Each forward-backward-update involves O(num_layer), which is often 100—1,000, tensor computations and communications

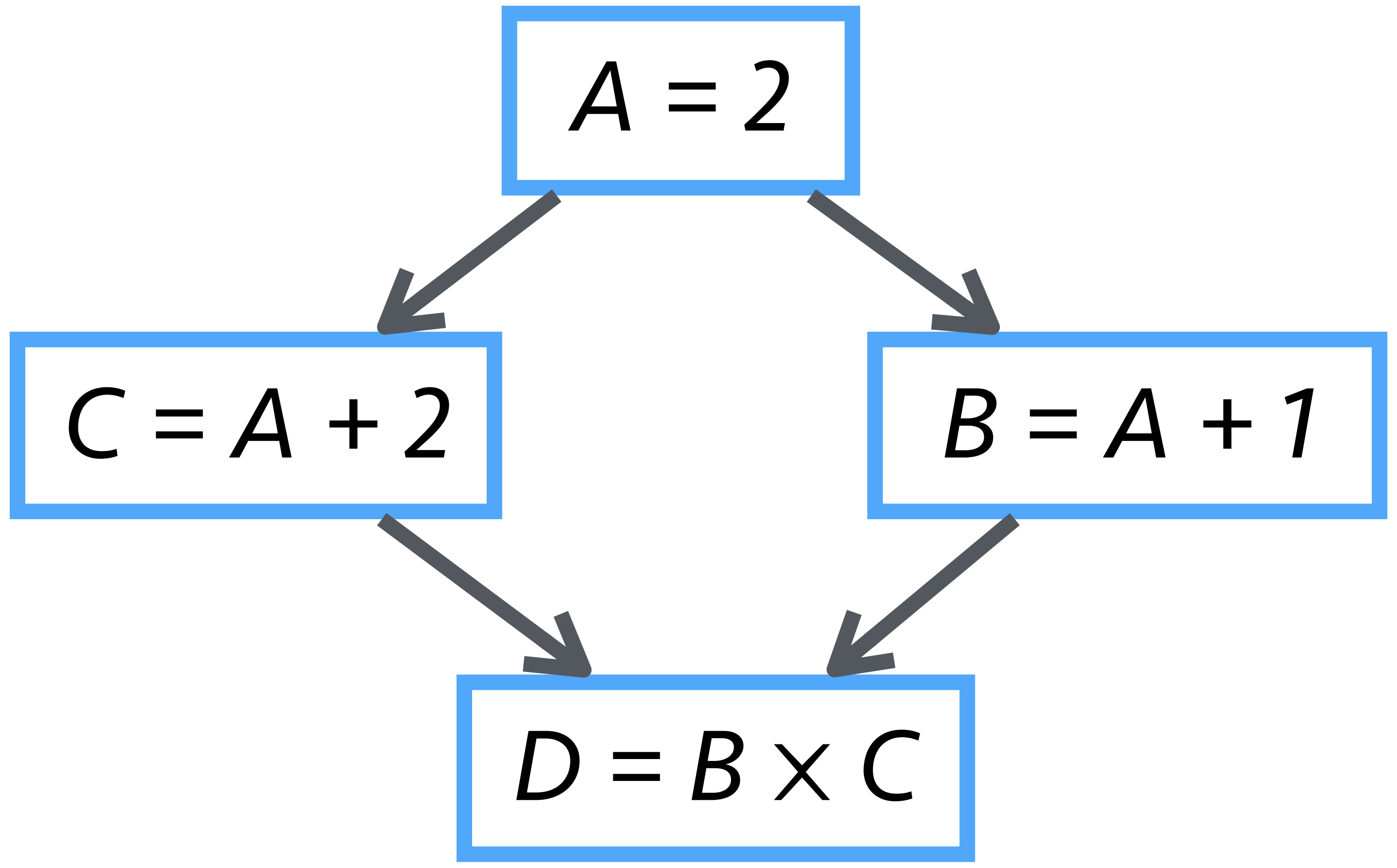


Auto Parallelization

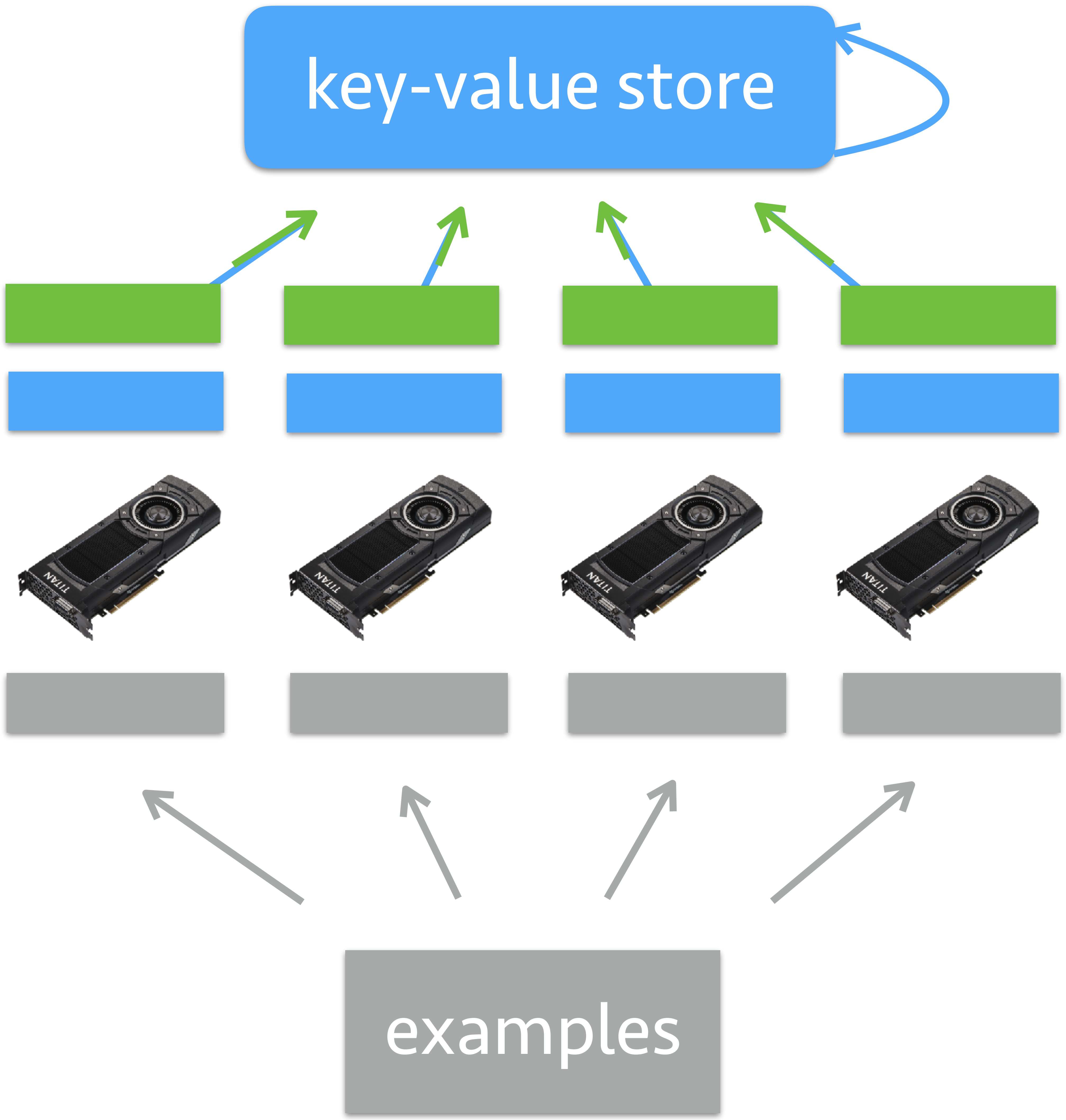
Write **serial** programs

```
>>> import mxnet as mx
>>> A = mx.nd.ones((2,2)) *2
>>> C = A + 2
>>> B = A + 1
>>> D = B * C
>>> D.wait_to_read()
```

Run in **parallel**



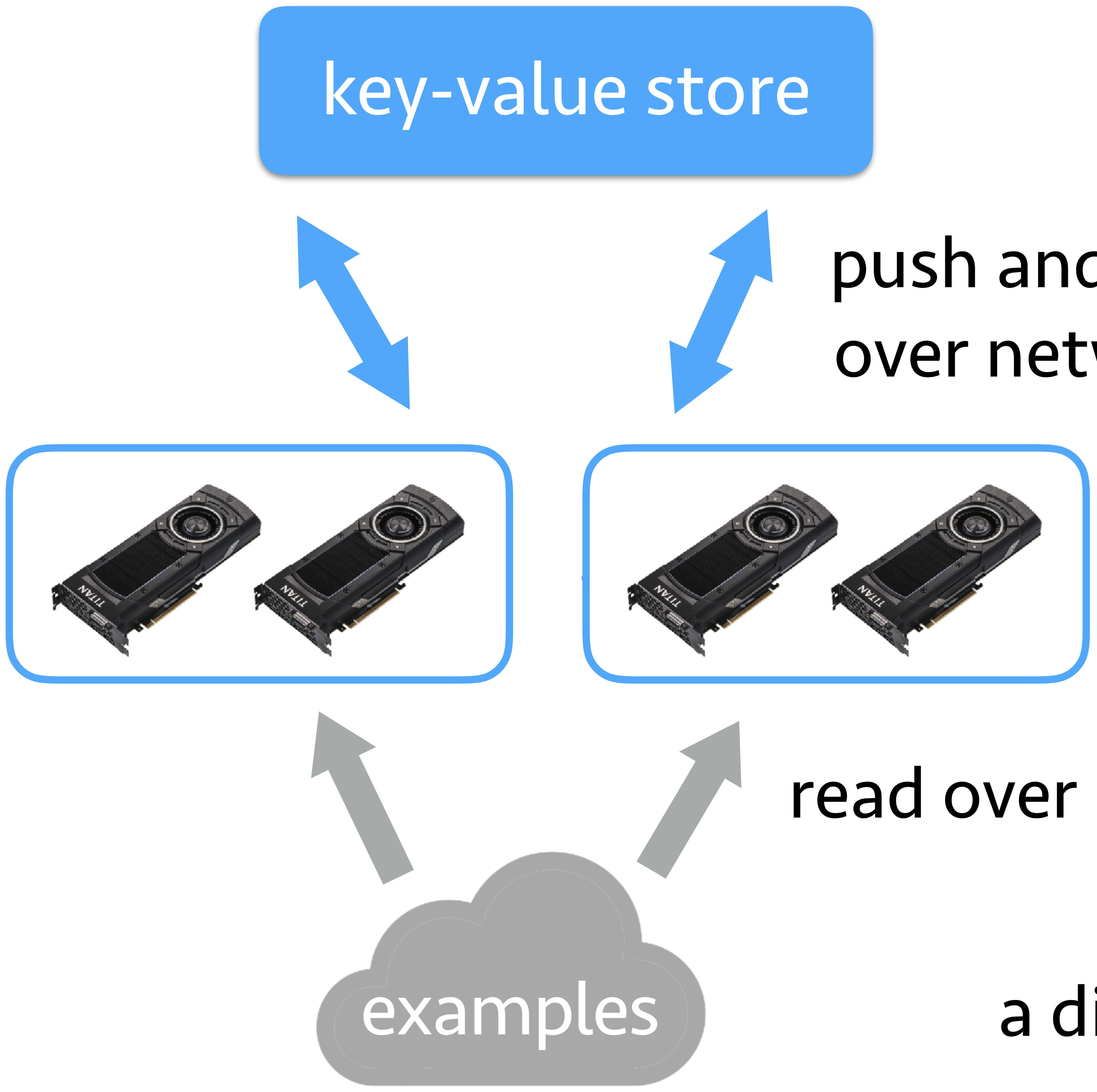
Data Parallelism



1. Read a data partition
2. Pull the parameters
3. Compute the gradient
4. Push the gradient
5. Update the parameters

Distributed Computing

A user does not need to change the codes when using multiple machines



multiple server machines

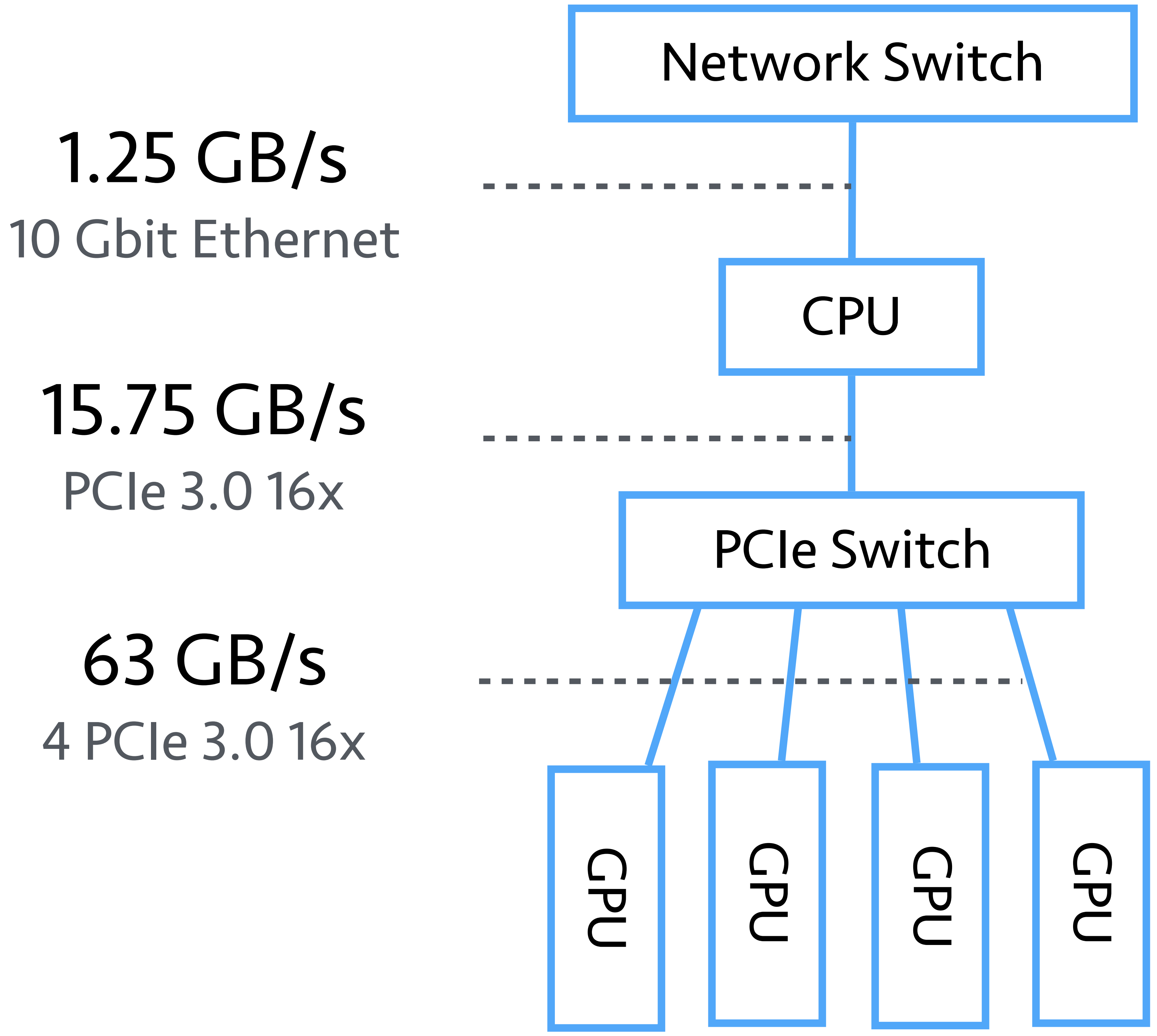
push and pull over network

multiple worker machines

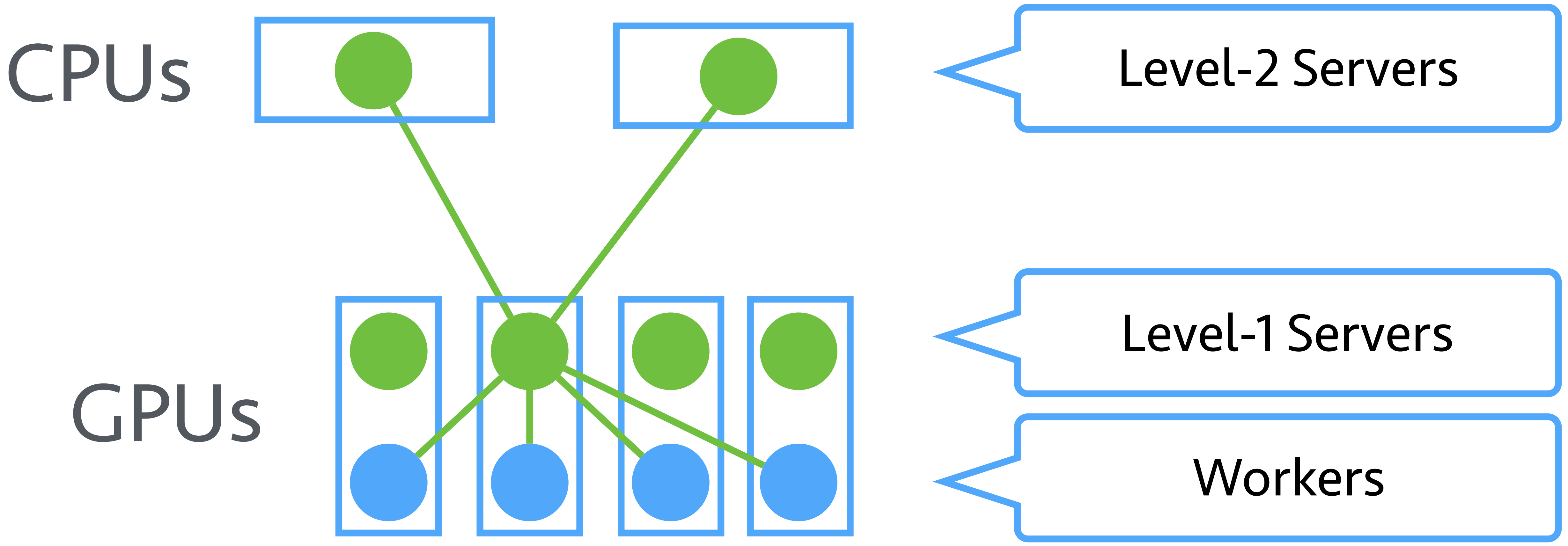
read over network

Store data in a distributed filesystem

Scale to Multiple GPU Machines



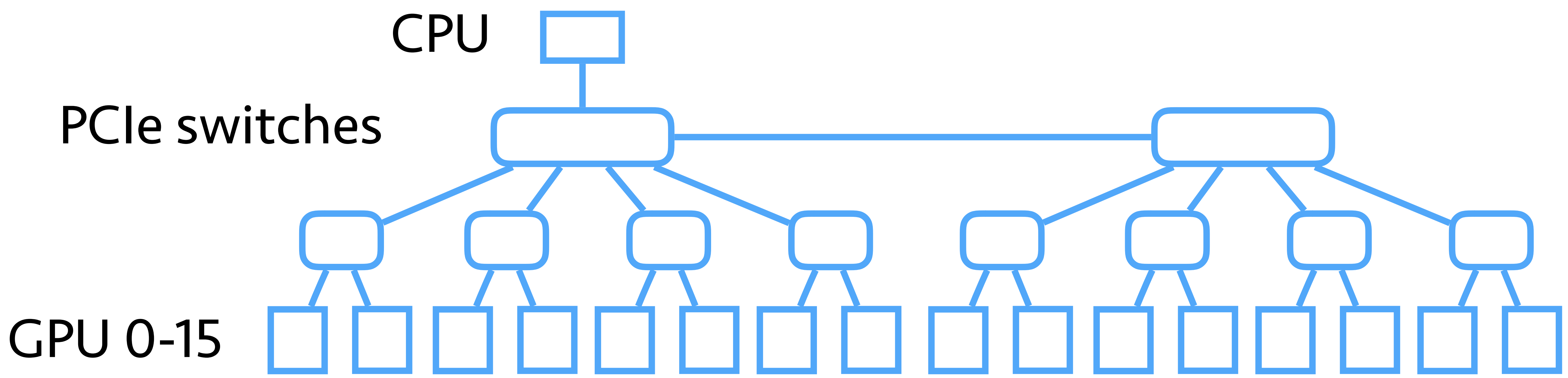
Hierarchical parameter server



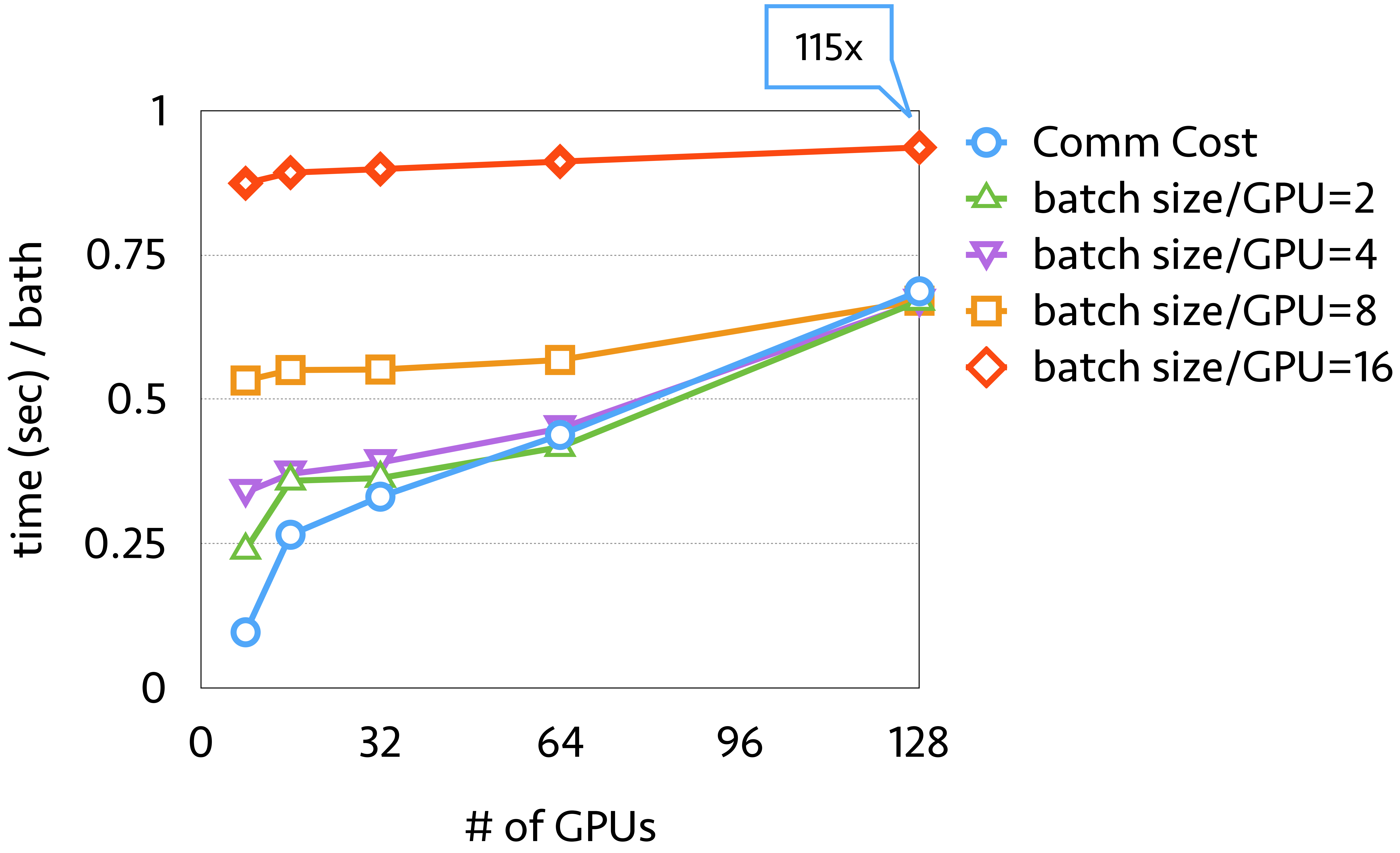
Experiment Setup

- IMAGENET
  - 1.2 million images with 1000 classes
- Resnet 152-layer model
- EC2 P2.16xlarge

- Minibatch SGD
- Synchronized Updating

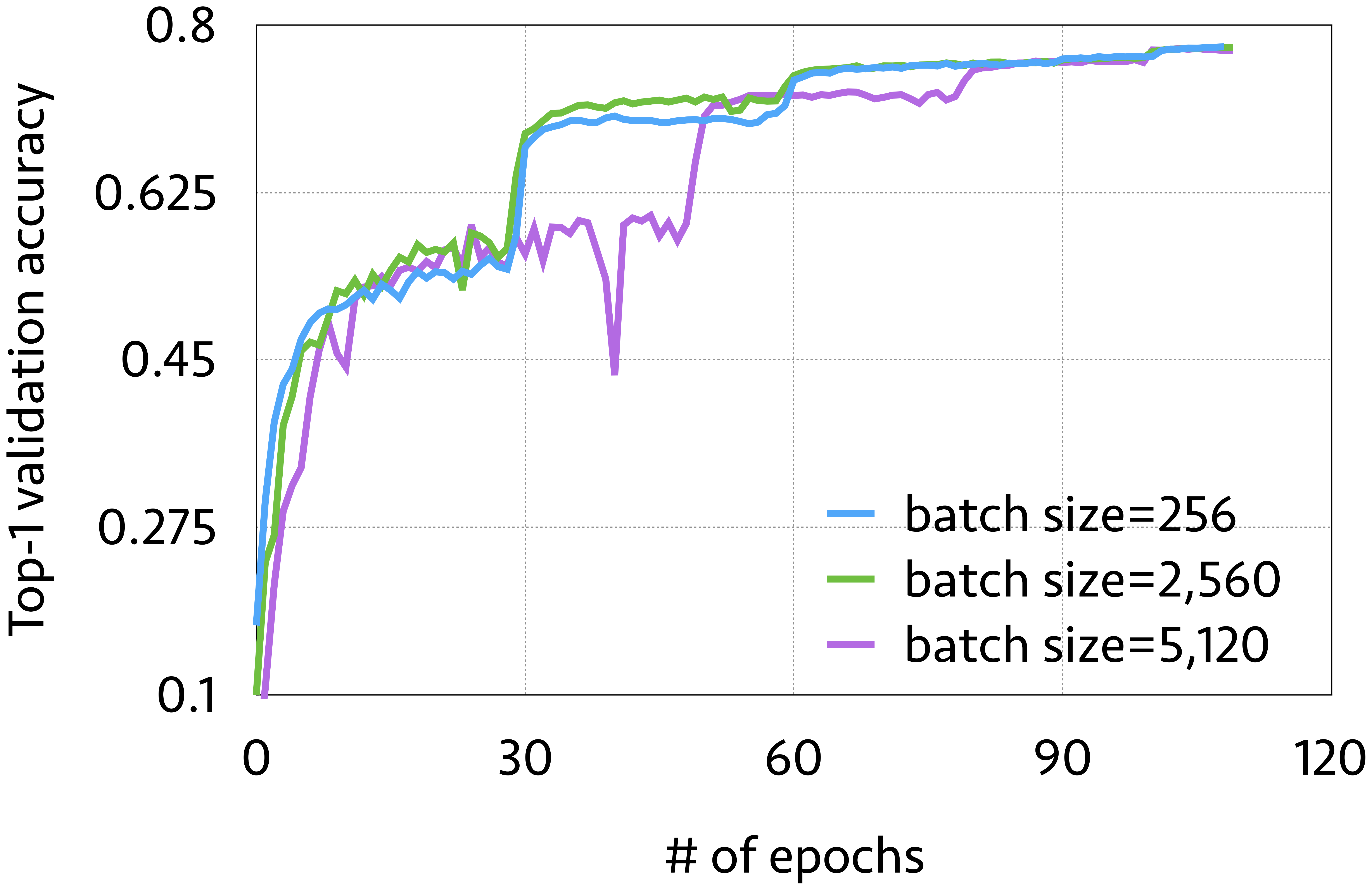


Scalability over Multiple Machines

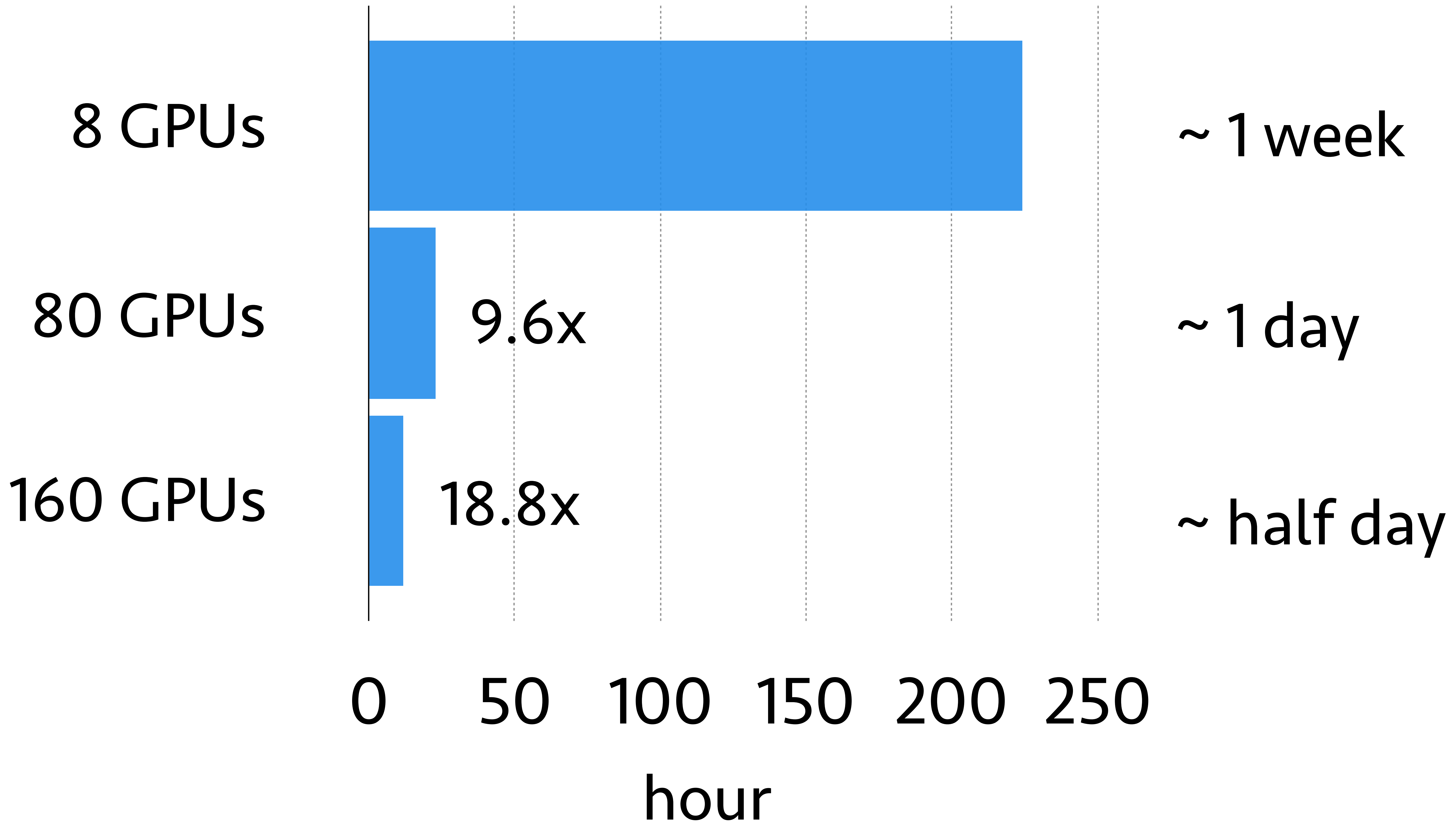


Convergence

- Increase learning rate by 5x
- Increase learning rate by 10x, decrease it at epoch 50, 80

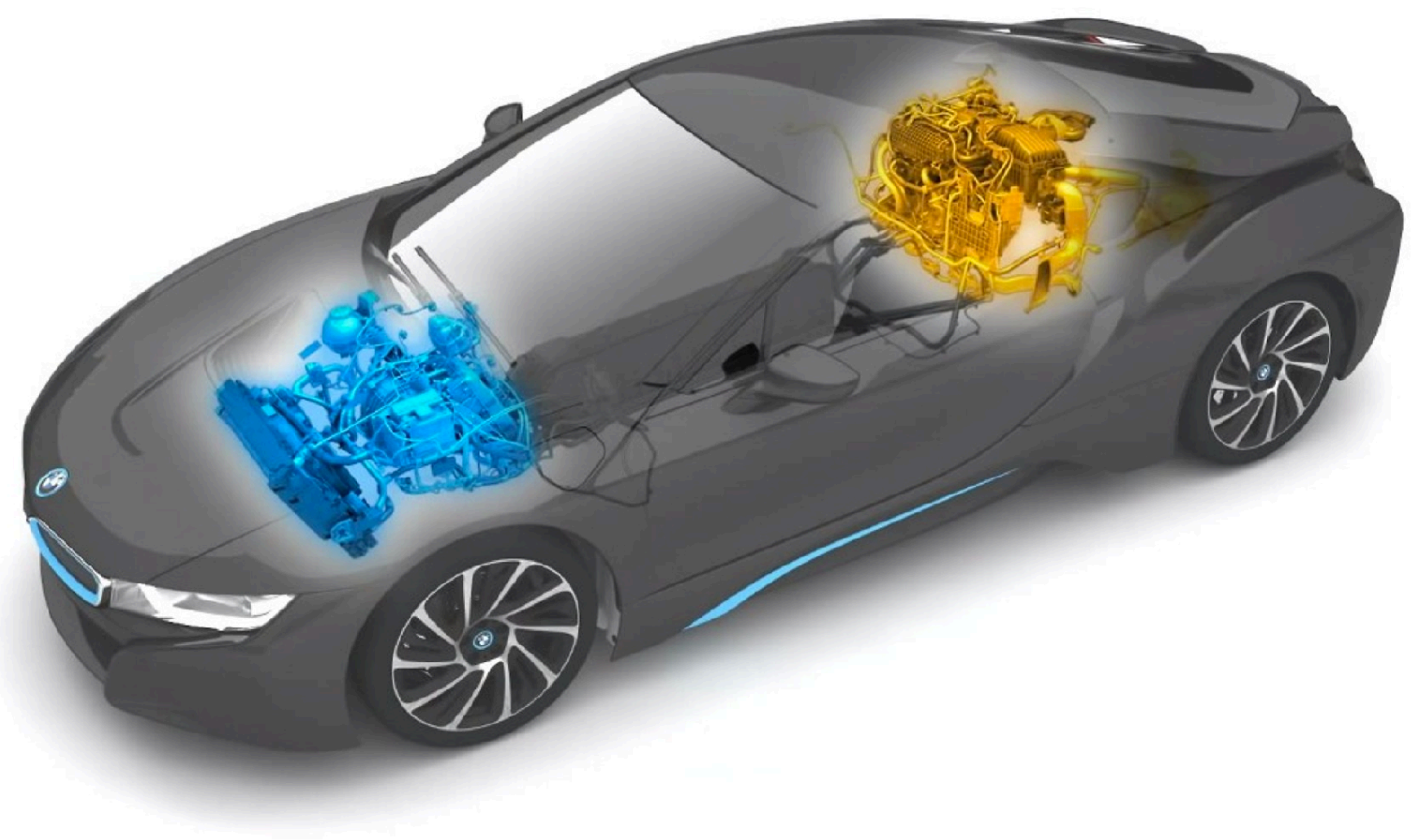


Time to achieve 22.5% top-1 accuracy



In summary

- Symbolic
  - efficient & portable
  - but hard to use
- Gluon
  - imperative for developing
  - symbolic for deploying
- Imperative
  - flexible
  - may be slow



Amazon Machine Image for Deep Learning

http://bit.ly/deepami

Getting started with Deep Learning

- For data scientists and developers
- Setting up a DL system takes (install) time & skill
  - Keep packages up to date and compile
  - Install all dependencies (licenses, compilers, drivers, etc.)
  - **NVIDIA** Drivers for G2 and P2 servers
  - **Intel** MKL linked for everything else (C5 coming soon)

amazon web services

http://bit.ly/deepami

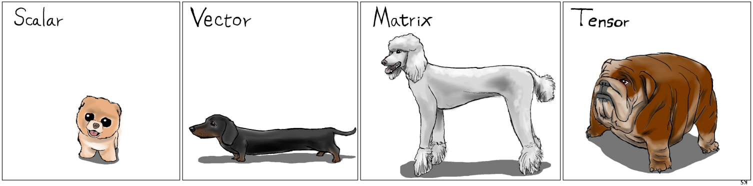
Deep Learning AMI

Sold by: Amazon Web Services

The Deep Learning AMI is a supported and maintained Amazon Linux image provided by Amazon Web Services for use on Amazon Elastic Compute Cloud (Amazon EC2). It is designed to provide a stable, secure, and high performance execution environment for deep learning applications running on Amazon EC2. It includes popular deep learning frameworks, including MXNet, Caffe, Tensorflow, Theano, Torch, and CNTK as well as packages that enable easy integration with AWS, including launch configuration tools and many popular AWS libraries and tools. It also includes the Anaconda Data Science Platform... Read more

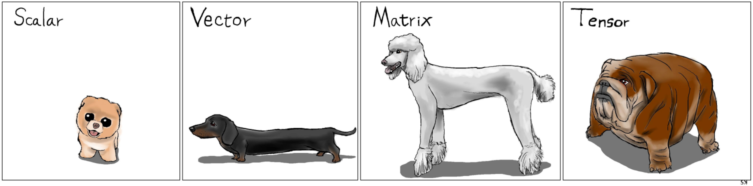
Outline

Tensors: Beyond 2D world



Modern data is inherently multi-dimensional

Tensors: Beyond 2D world



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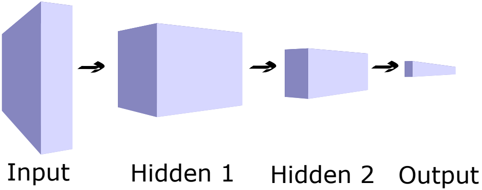
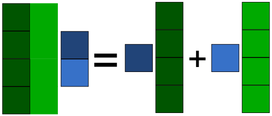


Tensor Contraction

Extends the notion of matrix product

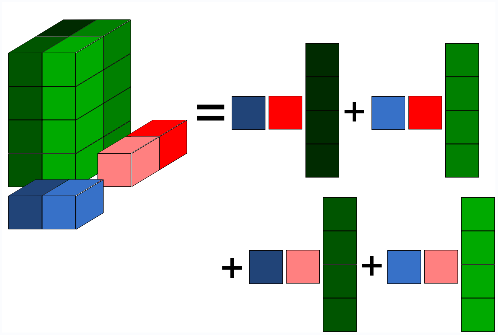
Matrix product

$$Mv = \sum_j v_j M_j$$

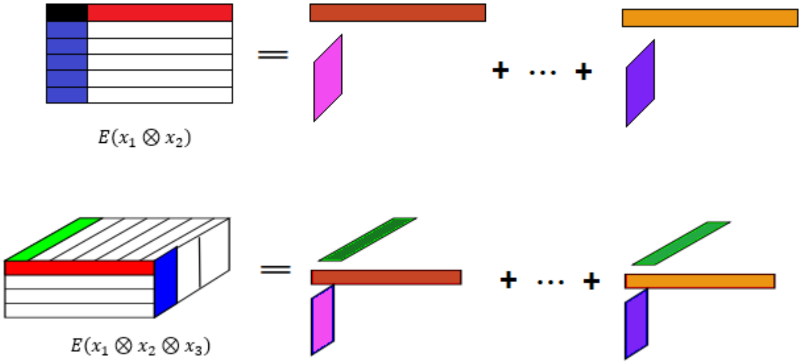


Tensor Contraction

$$T(u, v, \cdot) = \sum_{i,j} u_i v_j T_{i,j,:}$$

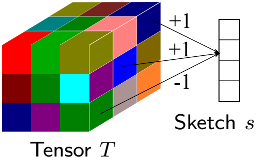


Tensor Decompositions



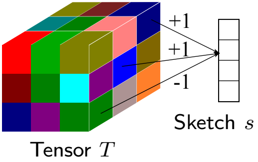
Tensor Sketches

- Dimensionality reduction through sketching.
  - Complexity independent of tensor order: exponential gain!



Tensor Sketches

- Randomized dimensionality reduction through sketching.
  - Complexity independent of tensor order: exponential gain!



Applications

- Tensor Decomposition via Sketching by Wang, Tung, Smola, A. NIPS'15.
- Compact Tensor Pooling for Visual Question and Answering by Shi, Anubhai, Furlanello, A, CVPR 2017 VQA workshop.

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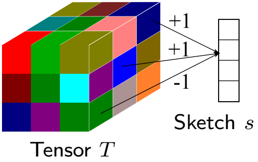

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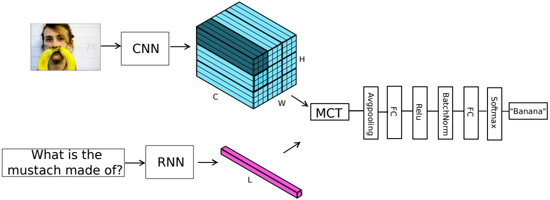
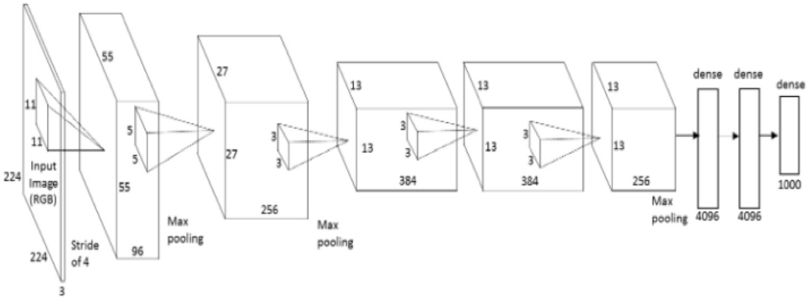
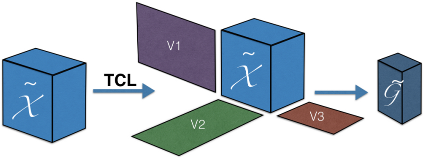


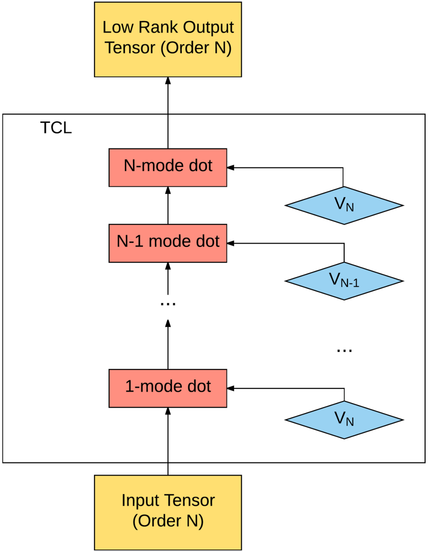
Employing Tensor Contractions in Alexnet



Replace fully connected layer with tensor contraction layer

Enabling Tensor Contraction Layer in Mxnet





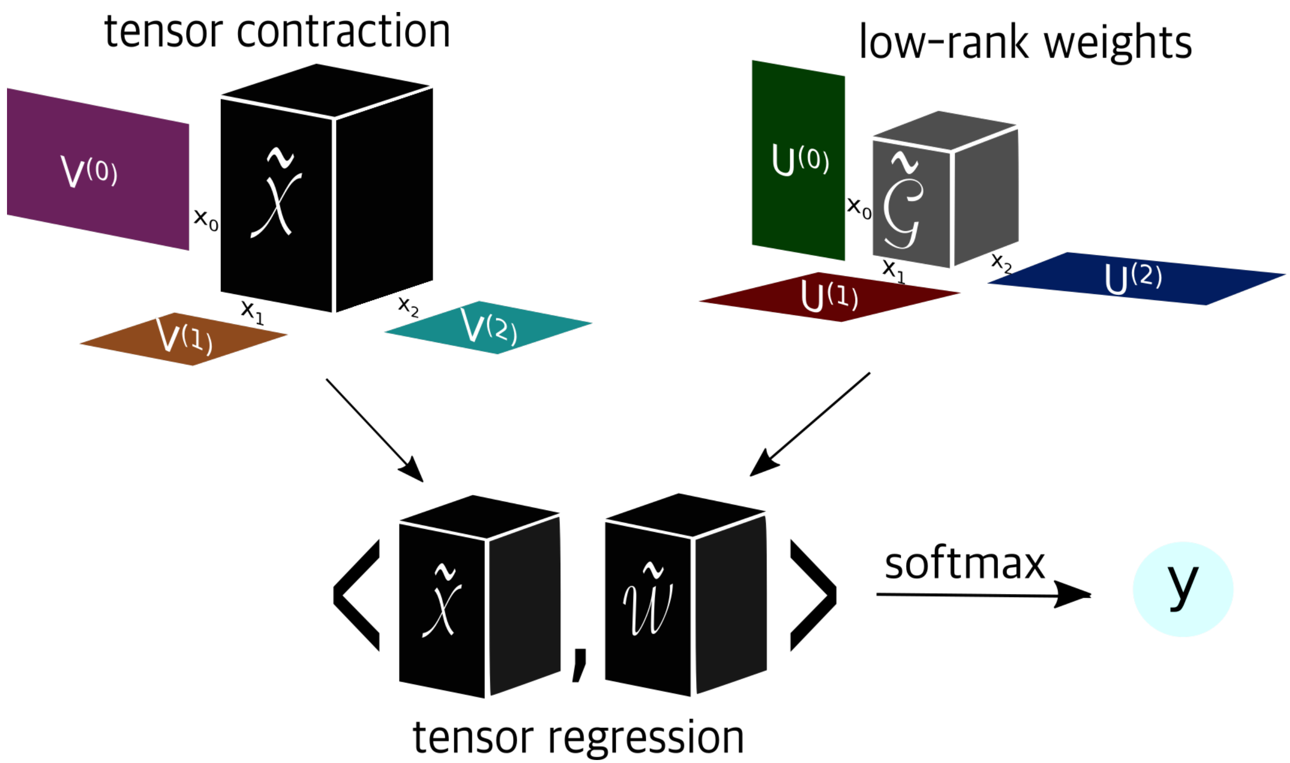
Performance of the TCL

- Trained end-to-end

- On ImageNet with VGG:
  - 65.9% space savings
  - performance drop of 0.6% only

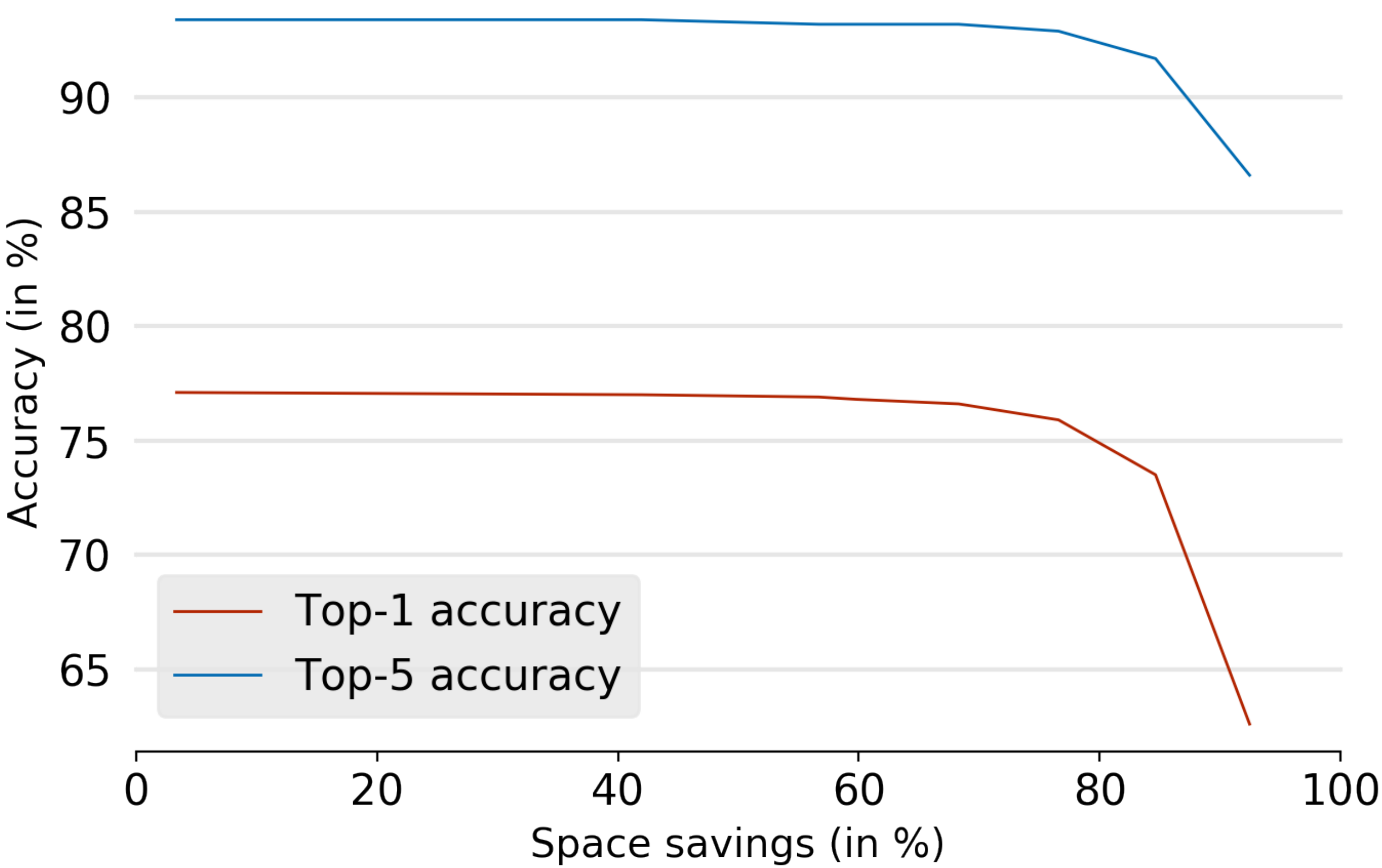
- On ImageNet with AlexNet:
  - 56.6% space savings
  - Performance improvement of 0.5%

Low-rank tensor regression



Tensor Regression Networks, *J. Kossaifi, Z.C.Lipton, A.Khanna, T.Furlanello and A.Anandkumar*, ArXiv pre-publication

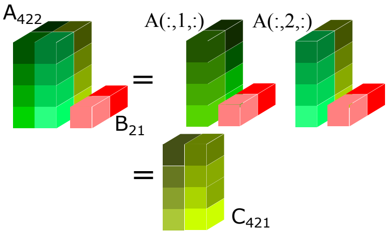
Performance and rank



Speeding up Tensor Contractions

1. Tensor contractions are a core primitive of multilinear algebra.
2. BLAS 3: Unbounded compute intensity (no. of ops per I/O)

Consider single-index contractions: $C_{\mathcal{C}} = A_{\mathcal{A}} \, B_{\mathcal{B}}$



e.g. $\quad C_{mnp} = A_{mnk} \, B_{kp}$

Speeding up Tensor Contractions

What do we have?

Tensor computation libraries

1. Arbitrary/restricted tensor operation of any order and dimension
    1. Tensortoolbox (Matlab)
    2. FTensor (C++)
    3. Cyclops (C++)
    4. BTAS (C++)
    5. All the Python...

Efficient computing frame

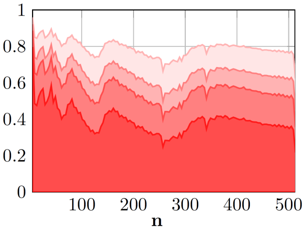
1. Static analysis solutions
    1. PPCG [ISL] (polyhedral)
    2. TCE (DSL)
2. Parallel and distributed primitives
    1. BLAS, cuBLAS
    2. BLIS, BLASX, cuBLASXT

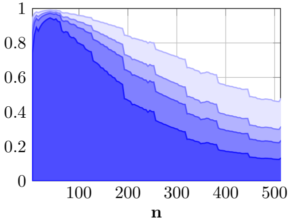
Speeding up Tensor Contraction

Explicit permutation dominates, especially for small tensors.

Consider $C_{mnp} = A_{km}\, B_{pkn}$.

1. $A_{km} \to A_{mk}$
2. $B_{pkn} \to B_{kpn}$
3. $C_{mnp} \to C_{mpn}$
4. $\boxed{C_{m(pn)} = A_{mk}\, B_{k(pn)}}$
5. $C_{mpn} \to C_{mnp}$





(Top) CPU. (Bottom) GPU. The fraction of time spent in copies/transpositions. Lines are shown with 1, 2, 3, and 6 transpositions.

Existing Primitives

GEMM

- Suboptimal for many small matrices.

Pointer-to-Pointer BatchedGEMM

- Available in MKL 11.3β and cuBLAS 4.1

$$C[p] = \alpha \, \mathsf{op}(A[p]) \, \mathsf{op}(B[p]) + \beta \, C[p]$$

```
cublas<T>gemmBatched(cublasHandle_t handle,
                     cublasOperation_t transA, cublasOperation_t transB,
                     int M, int N, int K,
                     const T* alpha,
                     const T** A, int ldA,
                     const T** B, int ldB,
                     const T* beta,
                     T** C, int ldC,
                     int batchCount)
```

Tensor Contraction with Extended BLAS Primitives

$$C_{mnp} = A_{**} \times B_{***}$$
$$C_{mnp} \equiv C[m + n \cdot \mathsf{ldC1} + p \cdot \mathsf{ldC2}]$$

Case	Contraction	Kernel1	Kernel2	Case	Contraction	Kernel1	Kernel2
1.1	$A_{mk}B_{knp}$	$C_{m(np)} = A_{mk}B_{k(np)}$	$C_{mn[p]} = A_{mk}B_{kn[p]}$	4.1	$A_{kn}B_{kmp}$	$C_{mn[p]} = B^\top_{km[p]}A_{kn}$	
1.2	$A_{mk}B_{kpn}$	$C_{mn[p]} = A_{mk}B_{k[p]n}$	$C_{m[n]p} = A_{mk}B_{kp[n]}$	4.2	$A_{kn}B_{kpm}$	$C_{mn[p]} = B^\top_{k[p]m}A_{kn}$	
1.3	$A_{mk}B_{nkp}$	$C_{mn[p]} = A_{mk}B^\top_{nk[p]}$		4.3	$A_{kn}B_{mkp}$	$C_{mn[p]} = B_{mk[p]}A_{kn}$	
1.4	$A_{mk}B_{pkn}$	$C_{m[n]p} = A_{mk}B^\top_{pk[n]}$		4.4	$A_{kn}B_{pkm}$		
1.5	$A_{mk}B_{npk}$	$C_{m(np)} = A_{mk}B^\top_{(np)k}$	$C_{mn[p]} = A_{mk}B^\top_{n[p]k}$	4.5	$A_{kn}B_{mpk}$	$C_{mn[p]} = B_{m[p]k}A_{kn}$	
1.6	$A_{mk}B_{pnk}$	$C_{m[n]p} = A_{mk}B^\top_{p[n]k}$		4.6	$A_{kn}B_{pmk}$		
2.1	$A_{km}B_{knp}$	$C_{m(np)} = A^\top_{km}B_{k(np)}$	$C_{mn[p]} = A^\top_{km}B_{kn[p]}$	5.1	$A_{pk}B_{kmn}$	$C_{(mn)p} = B^\top_{k(mn)}A^\top_{pk}$	$C_{m[n]p} = B^\top_{km[n]}A^\top_{pk}$
2.2	$A_{km}B_{kpn}$	$C_{mn[p]} = A^\top_{km}B_{k[p]n}$	$C_{m[n]p} = A^\top_{km}B_{kp[n]}$	5.2	$A_{pk}B_{knm}$	$C_{m[n]p} = B^\top_{k[n]m}A^\top_{pk}$	
2.3	$A_{km}B_{nkp}$	$C_{mn[p]} = A^\top_{km}B^\top_{nk[p]}$		5.3	$A_{pk}B_{mkn}$	$C_{m[n]p} = B_{mk[n]}A^\top_{pk}$	
2.4	$A_{km}B_{pkn}$	$C_{m[n]p} = A^\top_{km}B^\top_{pk[n]}$		5.4	$A_{pk}B_{nkm}$		
2.5	$A_{km}B_{npk}$	$C_{m(np)} = A^\top_{km}B^\top_{(np)k}$	$C_{mn[p]} = A^\top_{km}B^\top_{n[p]k}$	5.5	$A_{pk}B_{mnk}$	$C_{(mn)p} = B_{(mn)k}A^\top_{pk}$	$C_{m[n]p} = B_{m[n]k}A^\top_{pk}$
2.6	$A_{km}B_{pnk}$	$C_{m[n]p} = A^\top_{km}B^\top_{p[n]k}$		5.6	$A_{pk}B_{nmk}$		
3.1	$A_{nk}B_{kmp}$	$C_{mn[p]} = B^\top_{km[p]}A^\top_{nk}$		6.1	$A_{kp}B_{kmn}$	$C_{(mn)p} = B^\top_{k(mn)}A_{kp}$	$C_{m[n]p} = B^\top_{km[n]}A_{kp}$
3.2	$A_{nk}B_{kpm}$	$C_{mn[p]} = B^\top_{k[p]m}A^\top_{nk}$		6.2	$A_{kp}B_{knm}$	$C_{m[n]p} = B^\top_{k[n]m}A_{kp}$	
3.3	$A_{nk}B_{mkp}$	$C_{mn[p]} = B_{mk[p]}A^\top_{nk}$		6.3	$A_{kp}B_{mkn}$	$C_{m[n]p} = B_{mk[n]}A_{kp}$	
3.4	$A_{nk}B_{pkm}$			6.4	$A_{kp}B_{nkm}$		
3.5	$A_{nk}B_{mpk}$	$C_{mn[p]} = B_{m[p]k}A^\top_{nk}$		6.5	$A_{kp}B_{mnk}$	$C_{(mn)p} = B_{(mn)k}A_{kp}$	$C_{m[n]p} = B_{m[n]k}A_{kp}$
3.6	$A_{nk}B_{pmk}$			6.6	$A_{kp}B_{nmk}$		

Tensor Contraction with Extended BLAS Primitives

Case	Contraction	Kernel1	Kernel2	Kernel3
1.1	$A_{mk}B_{knp}$	$C_{m(np)} = A_{mk}B_{k(np)}$	$C_{mn[p]} = A_{mk}B_{kn[p]}$	$C_{m[n]p} = A_{mk}B_{k[n]p}$
6.1	$A_{kp}B_{kmn}$	$C_{(mn)p} = B^\top_{k(mn)}A_{kp}$	$C_{m[n]p} = B^\top_{km[n]}A_{kp}$	

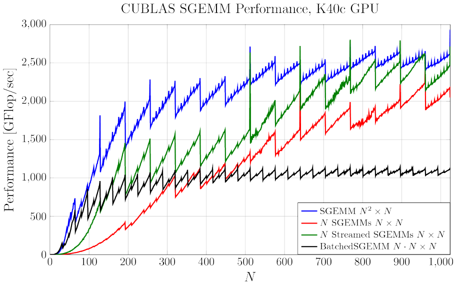
Example: Mappings to Level 3 BLAS routines

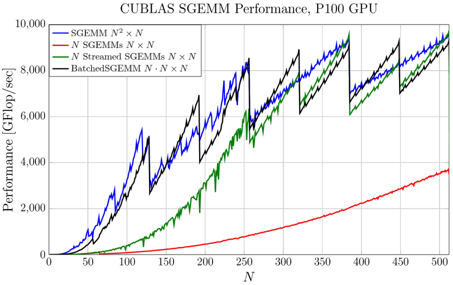
- Case 1.1, Kernel2: $C_{mn[p]} = A_{mk}B_{kn[p]}$

```
cublasDgemmStridedBatched(handle,
                          CUBLAS_OP_N, CUBLAS_OP_N,
                          M, N, K,
                          &alpha,
                          A, ldA1, 0,
                          B, ldB1, ldB2,
                          &beta,
                          C, ldC1, ldC2,
                          P)
```

Existing Primitives

Pointer-to-Pointer BatchedGEMM





A new primitive: StridedBatchedGEMM

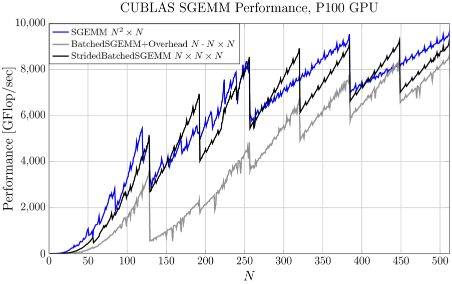
$$C[p] = \alpha \, \mathsf{op}(A[p]) \, \mathsf{op}(B[p]) + \beta \, C[p]$$

- Pointer-to-Pointer BatchedGEMM requires memory allocation and pre-computation.
- Solution: StridedBatchedGEMM with fixed strides.
  - Special case of Pointer-to-pointer BatchedGEMM.
  - No Pointer-to-pointer data structure or overhead.

```
cublas<T>gemmStridedBatched(cublasHandle_t handle,
                            cublasOperation_t transA, cublasOperation_t transB,
                            int M, int N, int K,
                            const T* alpha,
                            const T* A, int ldA1, int strideA,
                            const T* B, int ldB1, int strideB,
                            const T* beta,
                            T* C, int ldC1, int strideC,
                            int batchCount)
```

A new primitive: StridedBatchedGEMM

- Performance on par with pure GEMM (P100 and beyond).



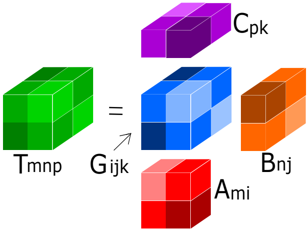
StridedBatchedGEMM

Documentation in cuBLAS 8.0:

```
$$ grep StridedBatched -A 17 /usr/local/cuda/include/cublas_api.h
2320:CUBLASAPI cublasStatus_t cublasSgemmStridedBatched (cublasHandle_t handle,
2321-                                                     cublasOperation_t transa,
2322-                                                     cublasOperation_t transb,
2323-                                                     int m,
2324-                                                     int n,
2325-                                                     int k,
2326-                                                     const float *alpha,  // host or device pointer
2327-                                                     const float *A,
2328-                                                     int lda,
2329-                                                     long long int strideA,   // purposely signed
2330-                                                     const float *B,
2331-                                                     int ldb,
2332-                                                     long long int strideB,
2333-                                                     const float *beta,   // host or device pointer
2334-                                                     float *C,
2335-                                                     int ldc,
2336-                                                     long long int strideC,
2337-                                                     int batchCount);
...
```

Applications: Tucker Decomposition

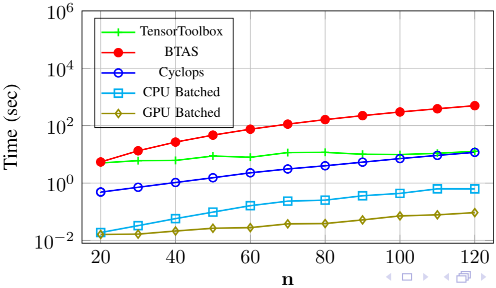
$$T_{mnp} = G_{ijk} A_{mi} B_{nj} C_{pk}$$



Main steps in the algorithm

- $Y_{mjk} = T_{mnp} B^t_{nj} C^t_{pk}$
- $Y_{ink} = T_{mnp} A^{t+1}_{mi} C^t_{pk}$
- $Y_{ijp} = T_{mnp} B^{t+1}_{nj} A^{t+1}_{mi}$

Performance on Tucker decomposition:



Applications: FFT

Low-Communication FFT for multiple GPUs involves tensor contractions.

$$
\begin{aligned}
T_{pib} &= S2T^{(p)}_{ijs}\, S_{pj(b+s)} &&\Longrightarrow && T_{pib} = S2T^{(p)}_{i(js)}\, S_{p(js)b} \\
M_{pqb} &= S2M_{qi}\, S_{pib} &&\Longrightarrow && M_{pq[b]} = S_{pi[b]}\, S2M^{T}_{qi} \\
M_{pqb'} &= M2M^{-}_{qm}\, M_{pmb^-} + M2M^{+}_{qm}\, M_{pmb^+} &&\Longrightarrow && M_{pq[b']} = M_{pM[b]}\, M2M^{T}_{qM} \\
r_p &= 1_{ib}\, S_{pib} = 1_{qb}\, M_{pqb} &&\Longrightarrow && r_p = 1_{(qb)}\, M_{p(qb)} \\
L_{pnb} &= M2L^{(p)}_{nms}\, M_{pm(b+s)} &&\Longrightarrow && L_{pnb} = M2L^{(p)}_{n(ms)}\, M_{p(ms)b} \\
L_{pqb^\pm} &= L2L^{\pm}_{qm}\, L_{pmb'} &&\Longrightarrow && L_{pq[b]} = L_{pM[b']}\, M2M_{qM} \\
T_{pib} &= L2T_{iq}\, L_{pqb} &&\Longrightarrow && T_{pi[b]} = L_{pq[b]}\, S2M_{qi}
\end{aligned}
$$

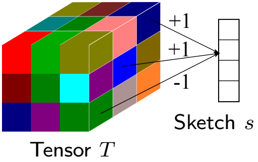
- StridedBatchedGEMM composes 75%+ of the runtime.
  - Essential to the performance.
  - Two custom kernels are precisely interleaved GEMMs.

- 2 P100 GPUs: 1.3x over cuFFTXT.
- 8 P100 GPUs: 2.1x over cuFFTXT.

C. Cecka "Low-Communication FFT with Fast Multipole Method" GTC 2017.

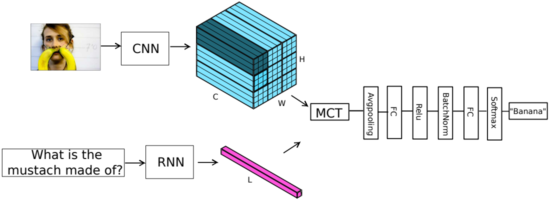
Tensor Sketches

- Randomized dimensionality reduction through sketching.
  - Complexity independent of tensor order: exponential gain!



Applications

- Tensor Decomposition via Sketching
- Visual Question and Answering

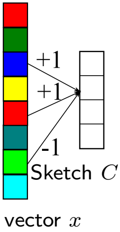


Tensor Sketching

- Dimensionality reduction through sketching.

Count Sketch for vector x

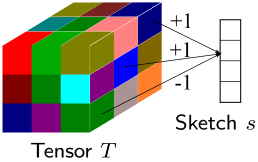
- $C[h[i]] += s[i]x[i]$, for $s[i] \in \{-1, +1\}^n$



Count Sketch for outer products $x \otimes y$

- Convolution of count sketches

$$C(x \otimes y, h, s) = C(x, h, s) * C(y, h, s)$$
$$= FFT^{-1}(FFT(C(x, h, s))FFT(C(y, h, s)))$$



Accelerated tensor low-rank decomposition

Symmetric tensor CP decomposition

For symmetric tensor $T \in \mathbb{R}^{n \times n \times n}$, find $\{(\lambda_i, u_i)\}_{i=1}^k$ to minimize

$$\|T - \sum_{i=1}^k \lambda_i u_i^{\otimes 3}\|_F^2.$$

- Wide application in data mining and latent variable models.
- **Tensor power iteration**: $u^{(t+1)} = T(I, u^{(t)}, u^{(t)}) / \|T(I, u^{(t)}, u^{(t)})\|_2$.
- Accelerated tensor power iteration via sketching:
  - TensorSketch: $s(T) \in \mathbb{R}^b$, for $n < b \ll n^3$.

$$\begin{aligned}[T(I,u,u)]_i &\approx \langle s(T), \quad s(u \otimes u \otimes e_i)\rangle \\ &= \langle \mathcal{F}(s(T)), \quad \mathcal{F}(s(u)). * \mathcal{F}(s(u)). * \mathcal{F}(s(e_i))\rangle \\ &= \langle \mathcal{F}^{-1}(\mathcal{F}(s(T)). * \overline{\mathcal{F}(s(u))}. * \overline{\mathcal{F}(s(u))}), \quad s(e_i)\rangle.\end{aligned}$$

  - Time complexity: $O(n^3) \to O(n + b \log b)$.

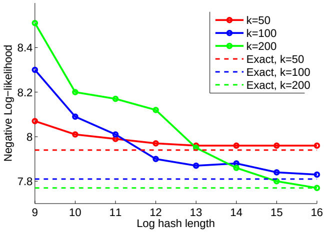
Efficient spectral method for topic modeling

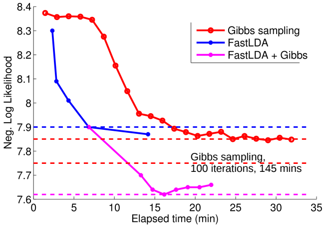
Topic modeling

V: vocabulary size; k: number of topics. Recover topic distributions $\mu_1, \cdots, \mu_k \in \mathbb{R}^V$ from N unlabeled documents.

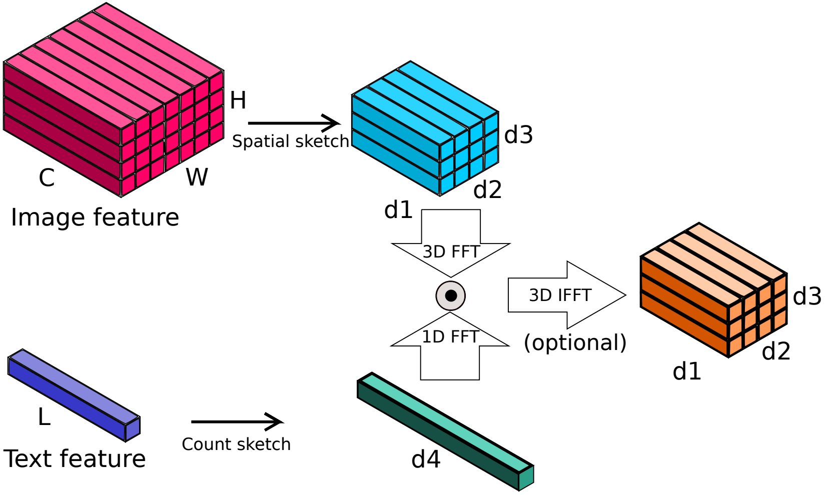
Figure 1: Negative log-likelihood and running time (min) on Wikipedia dataset.

k		like.	time	$\log_2 b$	iters	k	like.	time	$\log_2 b$	iters
200	Spectral	7.49	**34**	12	-	300	7.39	**56**	13	-
	Gibbs	6.85	561	-	30		6.38	818	-	30
	Hybrid	**6.77**	144	12	5		**6.31**	352	13	10

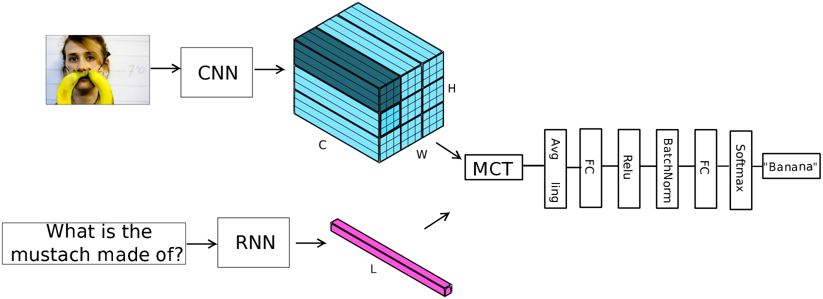




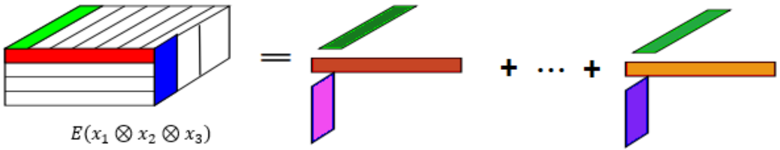
Multimodal Tensor Pooling



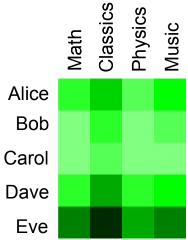
MCT in Visual Question & Answering

- 

Tensor Decompositions



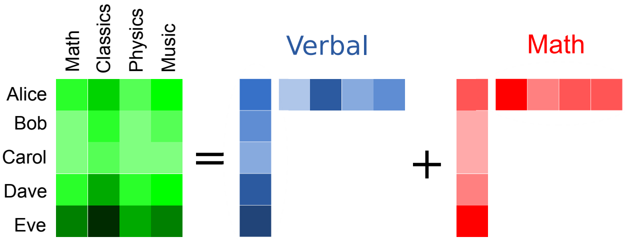
Example: Discovering Latent Factors



- List of scores for students in different tests
- Learn hidden factors for Verbal and Mathematical Intelligence [C. Spearman 1904]

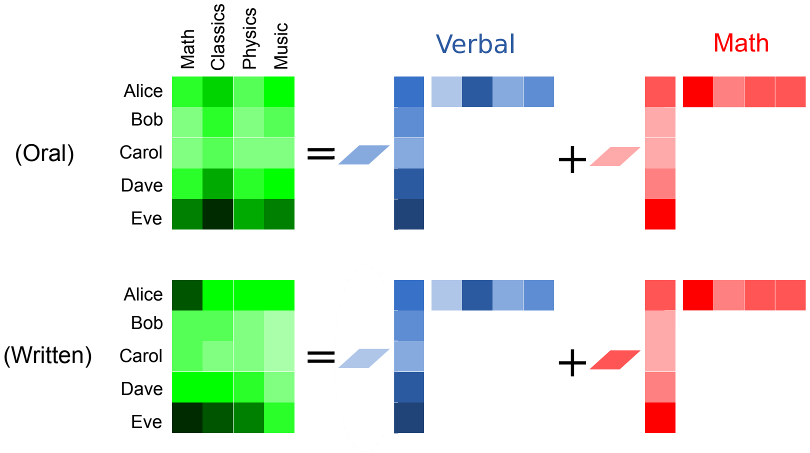
$$\text{Score (student,test)} = \text{student}_{\text{verbal-intlg}} \times \text{test}_{\text{verbal}} + \text{student}_{\text{math-intlg}} \times \text{test}_{\text{math}}$$

Matrix Decomposition: Discovering Latent Factors



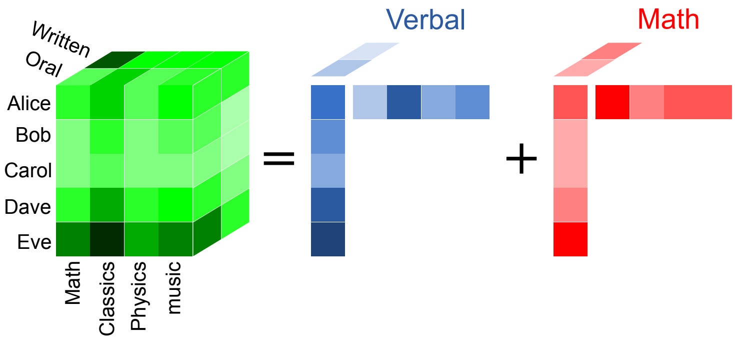
- Identifying hidden factors influencing the observations
- Characterized as matrix decomposition

Tensor: Shared Matrix Decomposition



- Shared decomposition with different scaling factors
- Combine matrix slices as a tensor

Tensor Decomposition



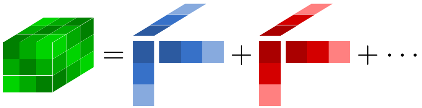
- Outer product notation:

$$T = u \otimes v \otimes w + \tilde{u} \otimes \tilde{v} \otimes \tilde{w}$$

$$\Updownarrow$$

$$T_{i_1,i_2,i_3} = u_{i_1} \cdot v_{i_2} \cdot w_{i_3} + \tilde{u}_{i_1} \cdot \tilde{v}_{i_2} \cdot \tilde{w}_{i_3}$$

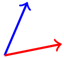
Identifiability under Tensor Decomposition



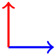
$$T = \lambda_1 a_1^{\otimes 3} + \lambda_2 a_2^{\otimes 3} + \cdots,$$

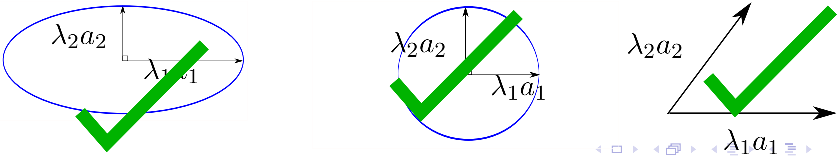
Uniqueness of Tensor Decomposition [J. Kruskal 1977]

- Above tensor decomposition: unique when rank one pairs are linearly independent



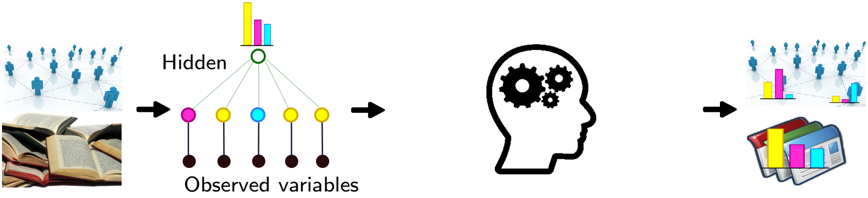
- Matrix case: when rank one pairs are orthogonal





Unsupervised Learning via Probabilistic Models

Data $\rightarrow$ Model $\rightarrow$ Learning Algorithm $\rightarrow$ Predictions



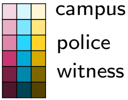
Challenges in High dimensional Learning

- Dimension of $x \gg$ dim. of latent variable h.
- Learning is like finding needle in a haystack.
- Computationally & statistically challenging.

Extracting Topics from Documents

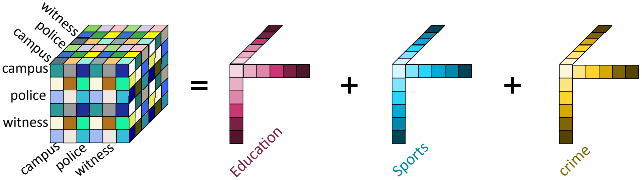
A., D. P. Foster, D. Hsu, S.M. Kakade, Y.K. Liu. "Two SVDs Suffice: Spectral decompositions for probabilistic topic modeling and latent Dirichlet allocation," NIPS 2012.

Tensor Methods for Topic Modeling



- Topic-word matrix $\boxed{\mathbb{P}[\text{word} = i | \text{topic} = j]}$
- Linearly independent columns

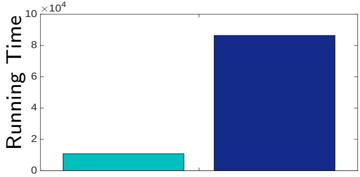
Moment Tensor: Co-occurrence of Word Triplets

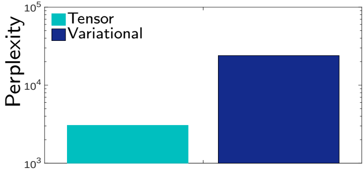


Tensors vs. Variational Inference

Criterion: Perplexity $= \exp[-\text{likelihood}]$.

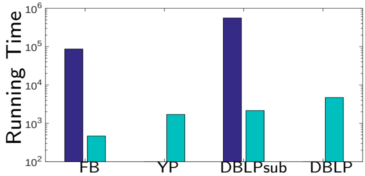
Learning Topics from PubMed on Spark, 8mil articles

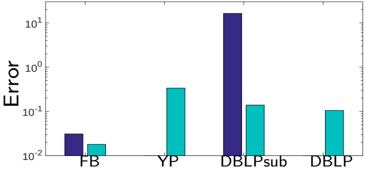




Learning network communities from social network data

Facebook $n \sim 20k$, Yelp $n \sim 40k$, DBLP-sub $n \sim 1e5$, DBLP $n \sim 1e6$.





F. Huang, U.N. Niranjan, M. Hakeem, A, "Online tensor methods for training latent variable models," JMLR 2014.

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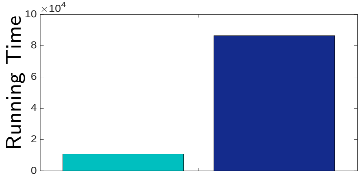
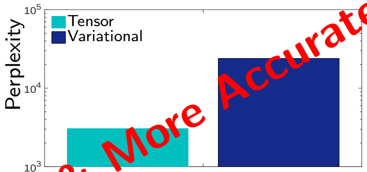
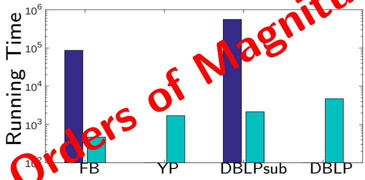
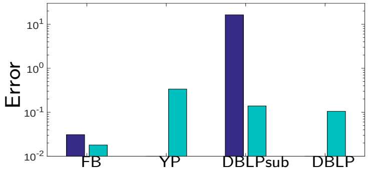




Learning network communities from social network data

Facebook $n \sim 20k$, Yelp $n \sim 40k$, DBLP-sub $n \sim 1e5$, DBLP $n \sim 1e6$.





Orders of Magnitude Faster & More Accurate

F. Huang, U.N. Niranjan, M. Hakeem, A, "Online tensor methods for training latent variable models," JMLR 2014.

TensorLy: Tensor Learning in Python

- Pure Python
- Integrated in the Python ecosystem
- Minimal dependencies (NumPy, SciPy and Matplotlib)
- Easy to use and extend
- Fast
- Extensively tested (unit-tests)
- Exhaustive documentation
- Open source, BSD licensed

Tensorly yours,

Try it: pip install tensorly

`https://tensorly.github.io`

Contributions welcome!

Outline

Conclusion

Distributed Deep Learning at Scale

- Mxnet has many attractive features
  - Flexible programming
  - Portable
  - Highly efficient
- Easy to deploy large-scale DL on AWS cloud
  - Deep Learning AMI
  - Cloud formation templates

Tensors are the future of ML

- Tensor contractions: space savings in deep architectures.

- New primitives speed up tensor contractions: extended BLAS