Reinforcement Learning: An Introduction

Deep Learning Indaba
September 2017

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UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

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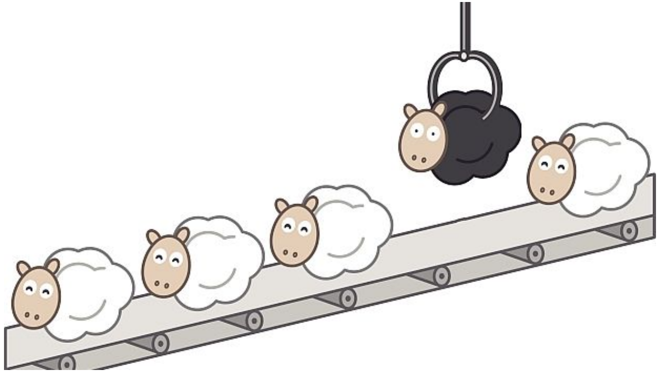
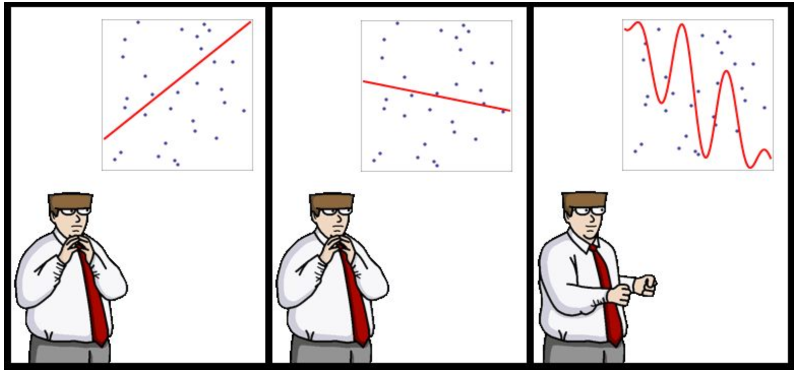
What is reinforcement learning?

We’ve seen how to solve many cool problems around supervised and unsupervised learning





But a major component of intelligence is decision making

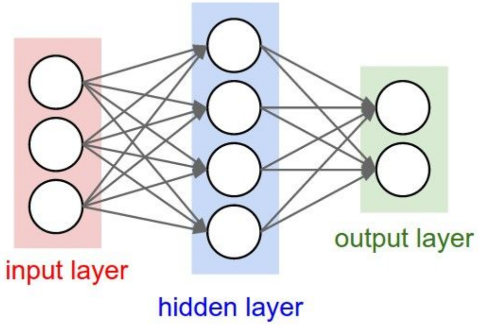
What is reinforcement learning?

Reinforcement learning is the branch of machine learning relating to learning in **sequential decision making settings**

Behaviour learning

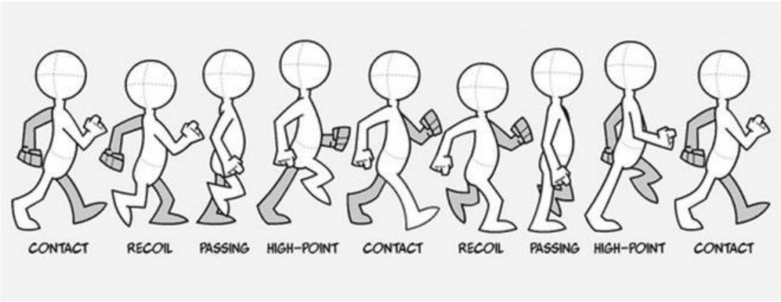
From supervised to reinforcement

Supervised learning, single decision point



Multiple decision points

- How do I know if I'm doing the right thing?
- How do my decisions now impact the future?
- Actions affect the environment!



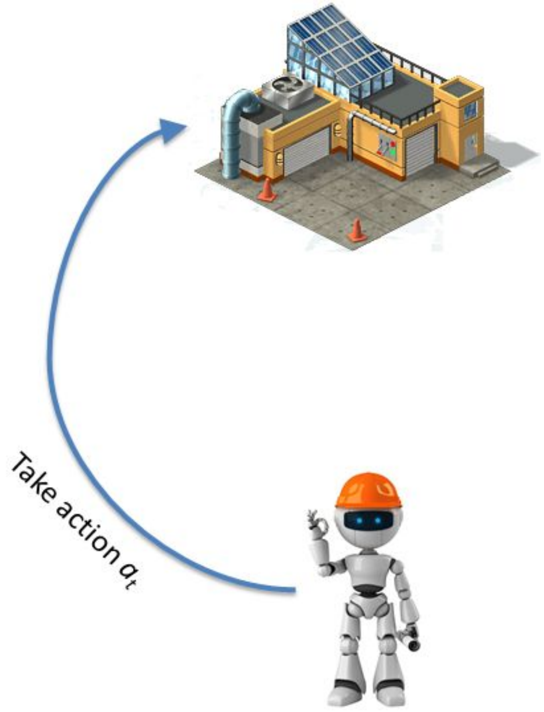
Interacting with an environment

Decision maker (agent) exists within an environment

Interacting with an environment

Decision maker (agent) exists within an environment

Agent takes **actions** based on the environment **state**



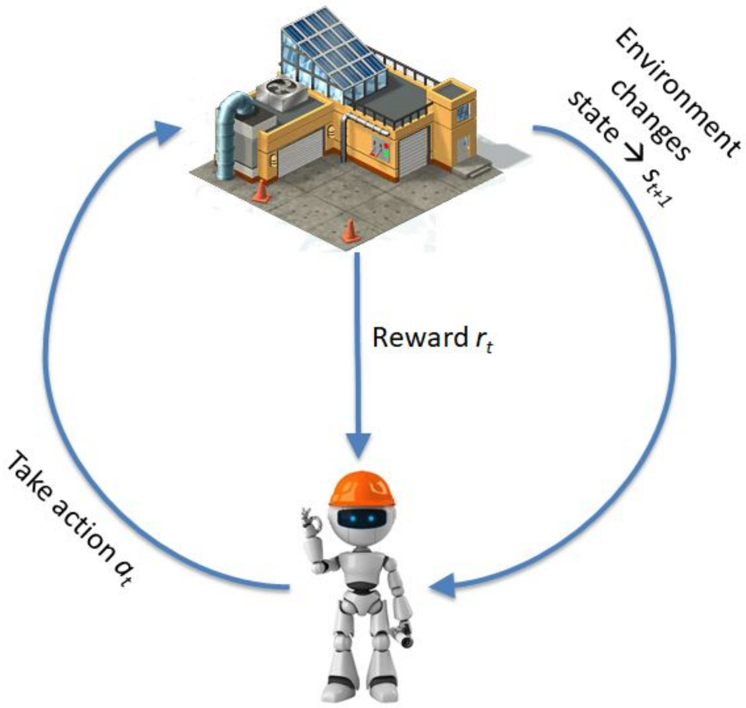
Interacting with an environment

Decision maker (agent) exists within an environment

Agent takes **actions** based on the environment **state**

Environment **state** updates

Agent receives feedback as **rewards**



A model for decision making

Markov Decision Process (MDP)

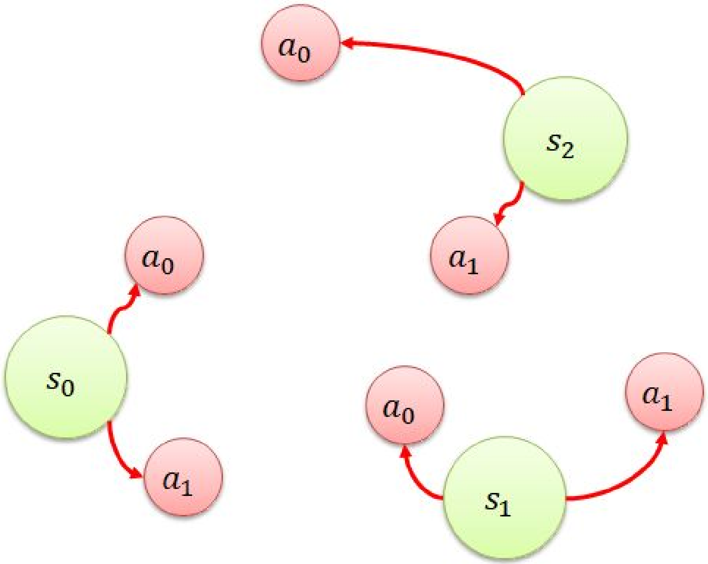
M = $\langle \boldsymbol{S}, \boldsymbol{A}, \boldsymbol{T}, \boldsymbol{R}, \gamma \rangle$

A model for decision making

Markov Decision Process (MDP)

M = $\langle \boldsymbol{S}, \boldsymbol{A}, \boldsymbol{T}, \boldsymbol{R}, \boldsymbol{\gamma} \rangle$

- States: encode world configurations
- Actions: choices made by agent



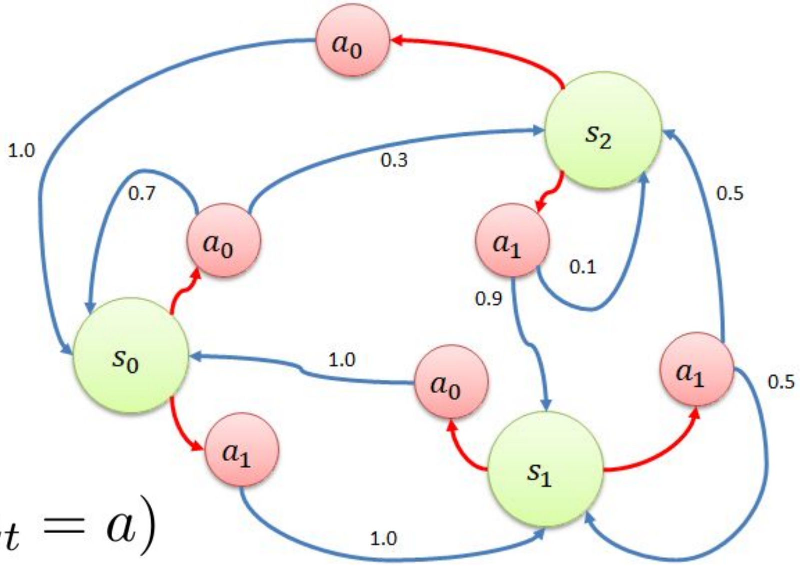
A model for decision making

Markov Decision Process (MDP)

M = ⟨S, A, T, R, γ⟩

Transition function: how the world evolves under actions

$$T(s, a, s') = P(s_{t+1} = s' | s_t = s, a_t = a)$$



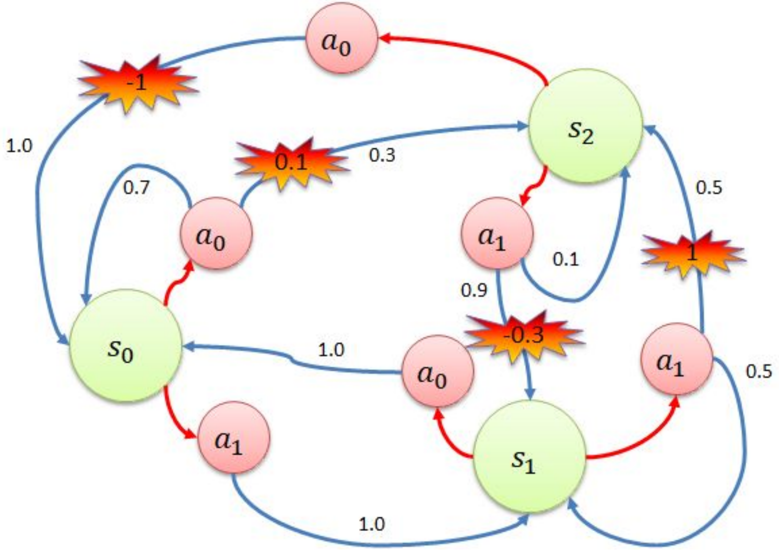
A model for decision making

Markov Decision Process (MDP)

M = $\langle \color{green}{S}, \color{red}{A}, \color{blue}{T}, \color{orange}{R}, \gamma \rangle$

Rewards: feedback signal to agent

$$R(s,a) = E[r_t | s_t = s, a_t = a]$$

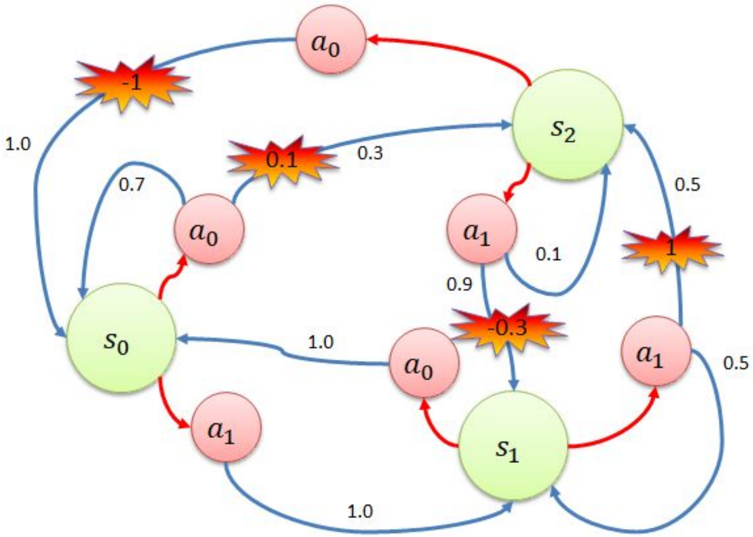


A model for decision making

Markov Decision Process (MDP)

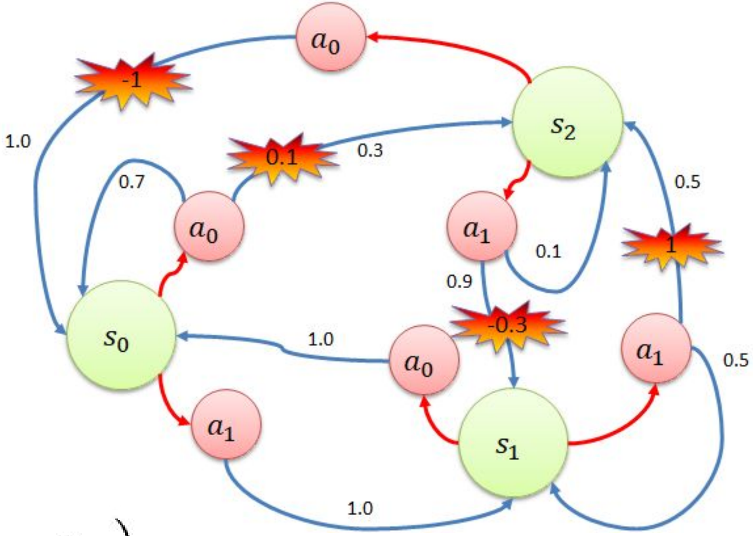
M = $\langle S, A, T, R, \gamma \rangle$

$\gamma \in [0,1]$ discounting for future rewards



A model for decision making

Markov Decision Process (MDP)



M = $\langle S, A, T, R, \gamma \rangle$

Markov:

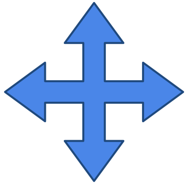
$$P(s_{t+1}|s_t) = P(s_{t+1}|s_0, ..., s_t)$$

"Future is independent of the past, given the present"

An example

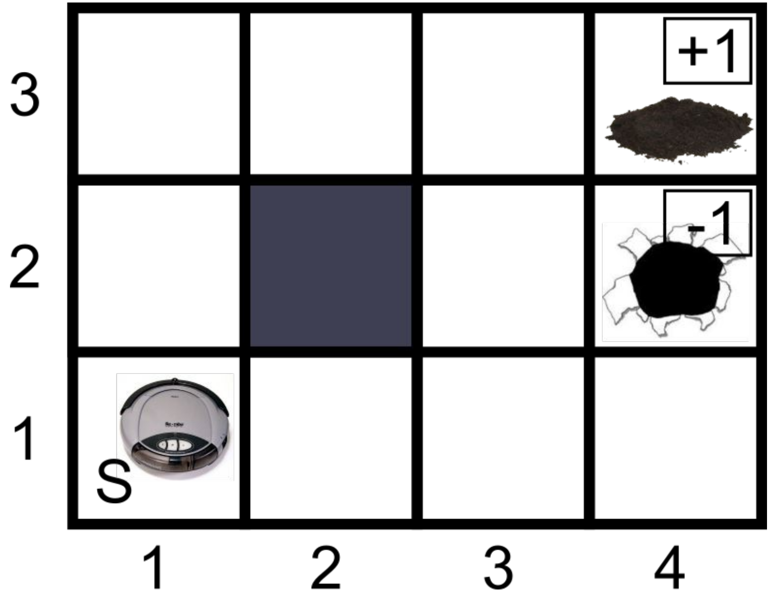
Cleaning Robot

Actions:



Reward:

- +1 for finding dirt
- -1 for falling into hole
- -0.001 for every move

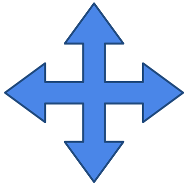


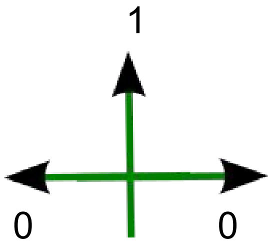
An example

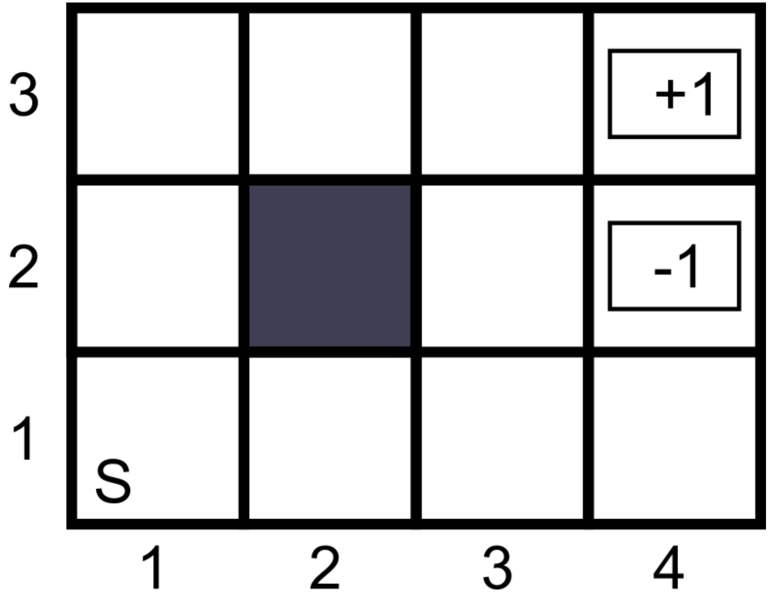
States:

- Position on grid e.g.
  - S is (1,1), goal (4,3)

Actions:







Reward:

- +1 for finding dirt
- -1 for falling into hole
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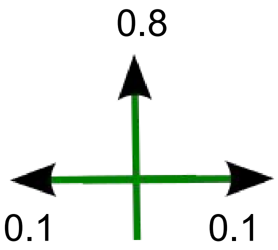
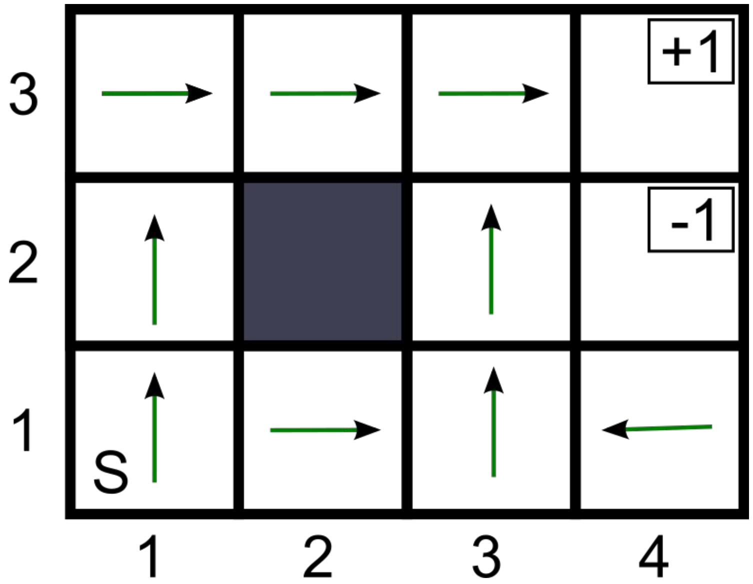
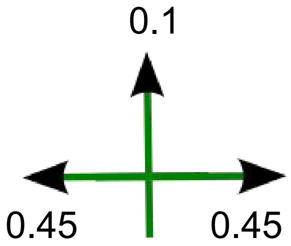
What is the optimal policy?





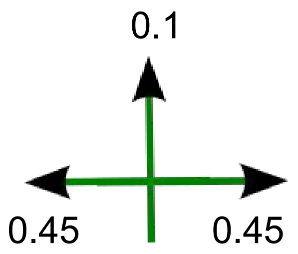
What is the optimal policy?

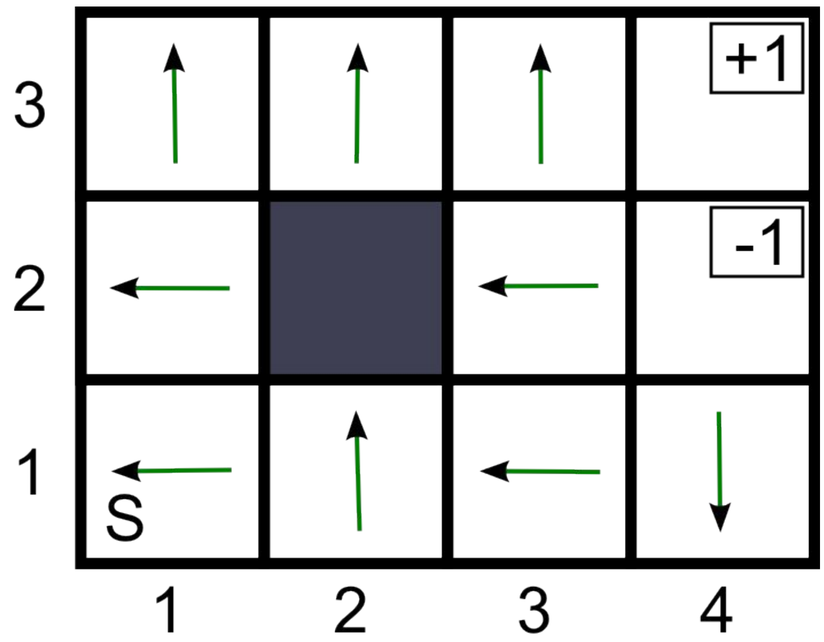
Change the action transitions?



What is the optimal policy?

Change the action transitions?



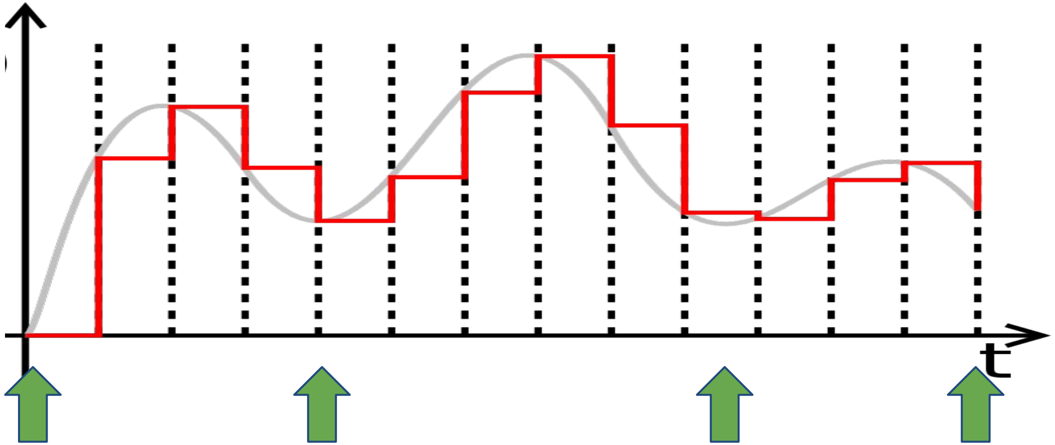


Practically, why RL?

- Treating disease in an individual
- Chronic disease (HIV, Cancer, Schizophrenia, etc.)
  - Not a single decision event

Information about:

- patient (demographics, family history)
- body (test results, etc.)
- disease (genomics, progression etc.)

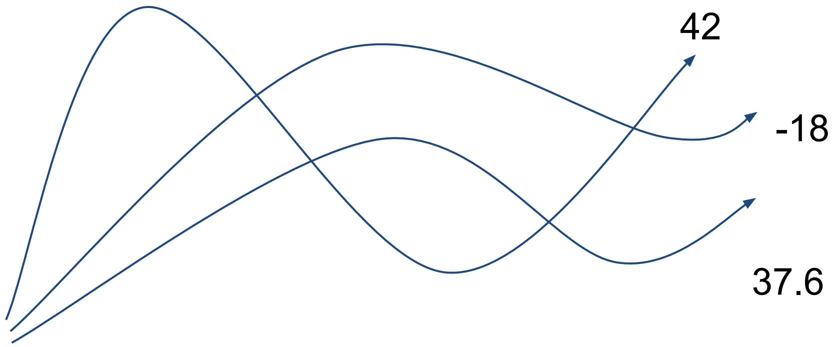


How do we find the best treatment strategy?

Evaluating behaviours

Many different trajectories
are possible through a space

Use the total **discounted accumulated rewards** to evaluate them



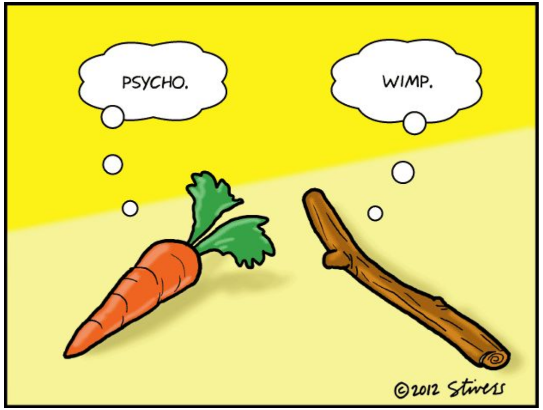
Rewards

Scalar feedback signal

Encode (un)desirable features of behaviours:

Winning/losing, collisions, taking expensive actions, ...

- Sparse
- Delayed
- Only have relative value



The Rats of Hanoi

Policies

A **policy** (or behaviour or strategy) π is any mapping from states to actions

- Deterministic or stochastic

$$\pi(a|s) = P(a_t = a | s_t = s)$$

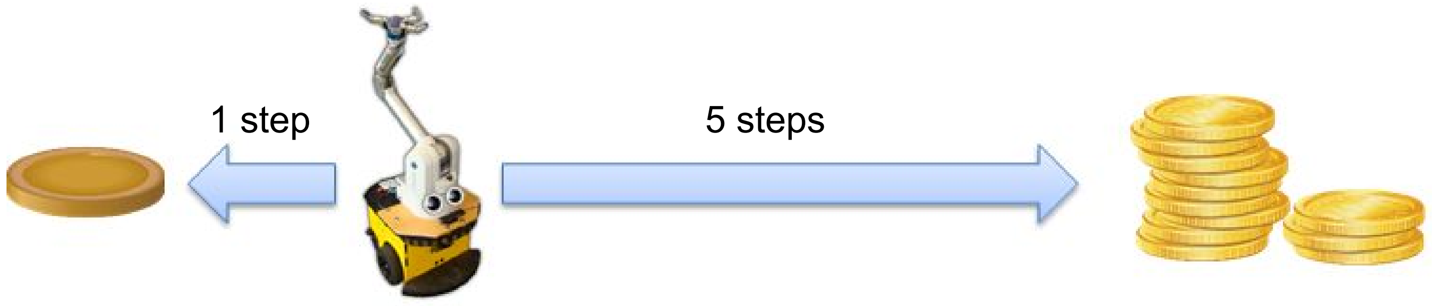
Optimal policy π^*

- Accumulates maximal rewards over a trajectory
- This is what we want to learn!

Immediate vs delayed rewards

Cannot just rely on the **instantaneous** reward function

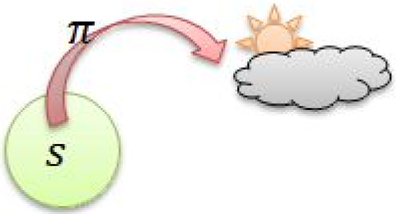
Tradeoff: don’t just act myopically (short term)



Notion of **value** to codify the goodness of a state, considering a policy running into the future

- Represented as a **value function**

Value Functions



Value function:

accumulated reward

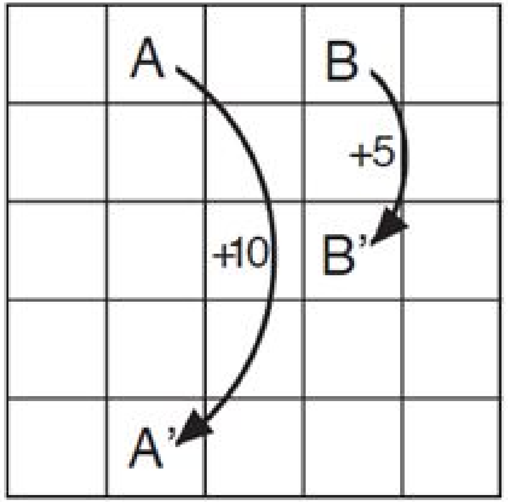
The expected return ($\boldsymbol{R}$) starting at state $\boldsymbol{s}$ and then executing policy $\boldsymbol{\pi}$

$$V^{\pi}(s) = E_{\pi}\{R_t | s_t = s\} = E_{\pi}\{\sum_{t=0}^{\infty} \gamma^t r_{\pi(s_t)}(s_t, s_{t+1})\}$$

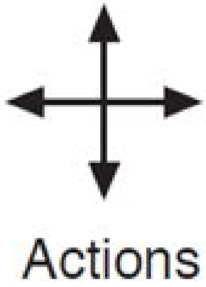
"How good is $\boldsymbol{s}$ under $\boldsymbol{\pi}$?"

Example Value Functions

Reward -1 for every move

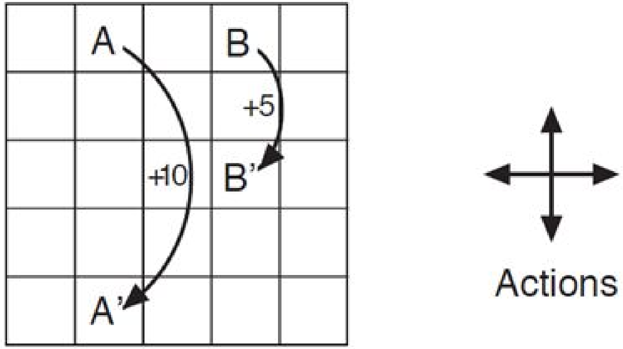


(a)



Example Value Functions

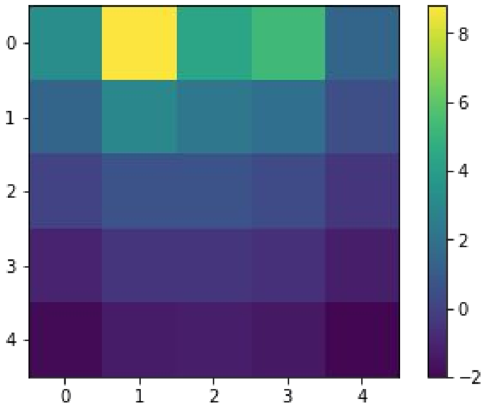
Random policy:



(a)

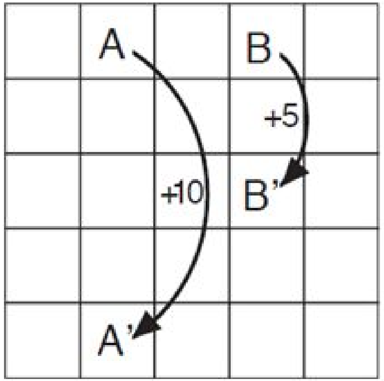
3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

(b)



Example Value Functions

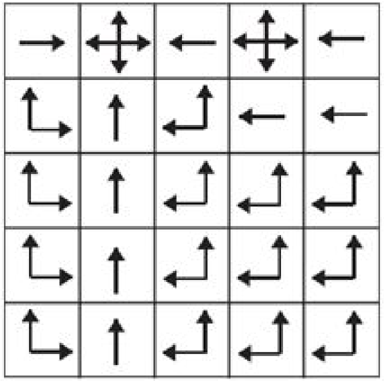
Optimal policy:



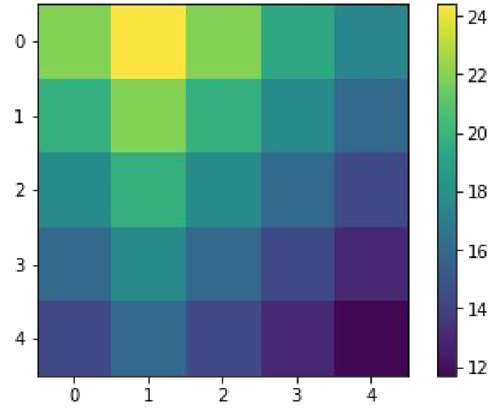
a) gridworld

22.0	24.4	22.0	19.4	17.5
19.8	22.0	19.8	17.8	16.0
17.8	19.8	17.8	16.0	14.4
16.0	17.8	16.0	14.4	13.0
14.4	16.0	14.4	13.0	11.7

b) v_*



c) π_*



So what?

How do we use these ideas
to do something useful?

Value Functions: Recursion

$V(s)$ ⇒ expected return starting at s and following π

Suggests dependence on V(s') from next state s'

Bellman Equation:

$$\boxed{V^\pi(s)} = \boxed{R(s,\pi(s))} + \gamma \boxed{\sum_{s'}} \boxed{T(s,\pi(s),s')} \boxed{V^\pi(s')}$$

value of s

immediate reward

for all possible next states

the probability of reaching that state with π

value of s'

Value Functions: Optimality

Similarly, for an optimal policy π^* with optimal value function V^*:

Bellman Optimality Equation:

$$\boxed{V^*(s)} = \boxed{\max_a}\{R(s,a) + \gamma \sum_{s'} T(s,a,s')\boxed{V^*(s')}$$

take the best possible action

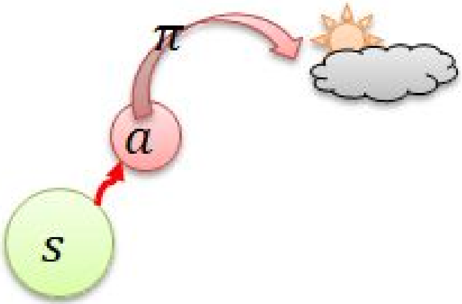
Value Functions

Action-value function:

$$\boxed{Q(s,a)} = \sum_{s'} \underbrace{\boxed{T(s,a,s')}}_{\text{transition probability}} \left(R(s,a,s') + \gamma \boxed{V(s')}\right)$$

The expected return (***R***) starting at state ***s*** and executing action ***a***, and then following policy ***π***

“How good is ***a*** in ***s*** under ***π***?”



Optimal policies and value functions

$\pi^*(a|s) :=$ $\quad$ 1 if $a = argmax\ Q^*(s,a)$,
0 otherwise

Move in direction of greatest value

Finding Q^* (or V^*) is equivalent to finding π^*

Every MDP has an optimal policy

The goal of RL

Given this formulation,

how do we learn a policy?

Solving Bellman

Given the Bellman equation

$$V^*(s) = \max_a \{R(s,a) + \gamma \sum_{s'} T(s'|s,a)V^*(s')\}$$

Solve this as a large system of value function equations

- But: non-linear (max operator)
- So: solve iteratively

What are we trying to do here?

- Learn how good each state of the world is, when looking perfectly into the future

Dynamic Programming

Value Iteration: Dynamic Programming

(T,R,S,A)

- Iteratively update V (synchronous version)
- At each iteration ***i***:
  - For all states ***s*** in ***S***:
    - Update ***V(s)***

$$V_{i+1}(s) := \max_a \left\{ \sum_{s'} T(s,a,s') \left(R(s,a,s') + \gamma V_i(s') \right) \right\}$$

But: this requires the full MDP!!

In general, T and R are unknown

Value Based Methods

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Algorithm setup

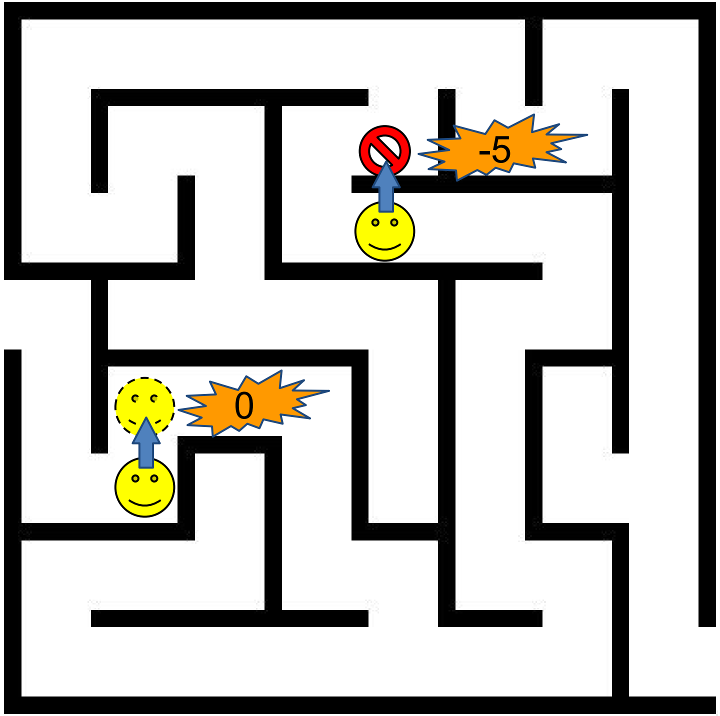
Value Based Methods:

- No Transition Model
- No Reward Model
- Access to environment for experiment or access to training data ***(s,a,r,s')***
- **Goal:** Learn Value of States, State-Actions
  - Policy through learned values

Data generation

T and R unknown!

Instead, generate samples of training data ***(s,a,r,s')*** from environment

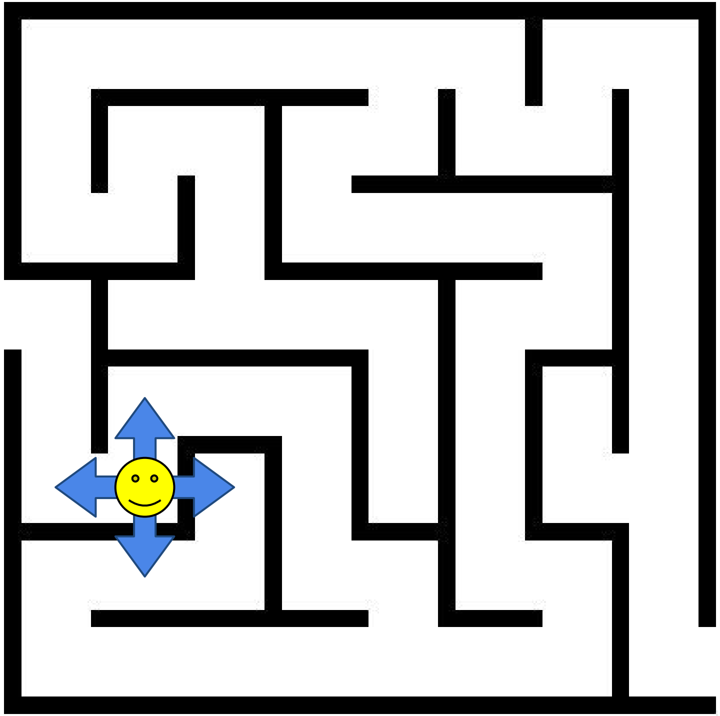


Learning from Experience

We need

- **A method to choose actions**

- Some model to keep track of and learn
  - *Value Function*



The Bandit Problem

Consider a row of one-arm bandit machines in a casino

Set of “arms” (actions) that each generate rewards from different distributions

Exploration vs exploitation

Action selection

The exploration-exploitation tradeoff!

Maximizing expected returns means balancing between:

- ***Exploiting*** gained knowledge (greedy)
  - Take the best known action
- ***Exploring*** new actions/states (random)
  - Try something new

Action selection strategies

ε-Greedy ($0 < \varepsilon \leq 1$):

- With probability $1-\varepsilon$ exploit
  - Choose the best action for a state
- With probability ε explore
  - Randomly choose action

ε usually higher at beginning of learning, decay later

Softmax

- Sample action given softmax

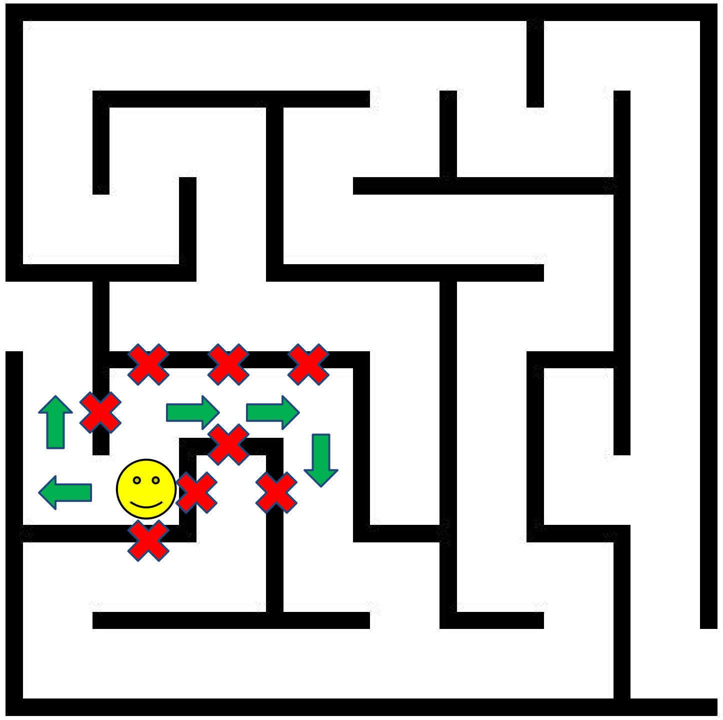
$$P_t(a) = \frac{\exp(Q_t(a)/\tau)}{\sum_{i=1}^{n} \exp(Q_t(i)/\tau)},$$

Learning from Experience

We need

- A method to choose actions

- **Some model to keep track of and learn**
  - ***Value Function***



TD Learning

(*T*,*R*,*S*,*A*)

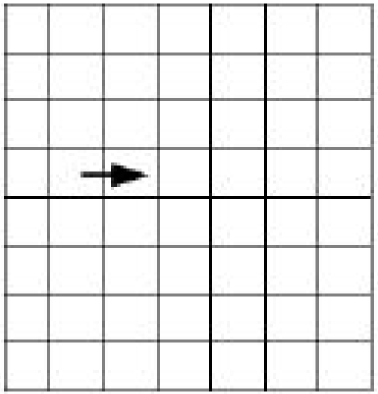
Temporal Difference (TD) Learning:

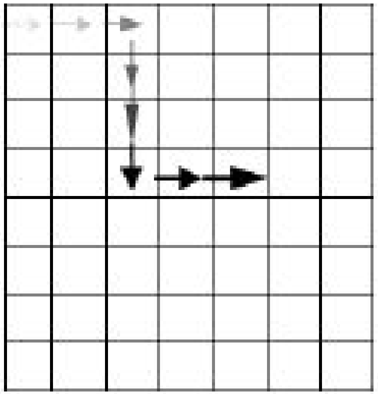
- Initialise ***V*** for all ***s*** in ***S***
- For each experience tuple (***s***,***r***,***s'***) under policy ***π***:
  - Update ***V***: $$V_{i+1}(s) \leftarrow V_i(s) + \alpha(\underbrace{\overbrace{r + \gamma V_i(s')}^{\textit{learnt value}} - V_i(s)}_{\text{TD error}})$$

estimated return
(TD target)

Eligibility traces

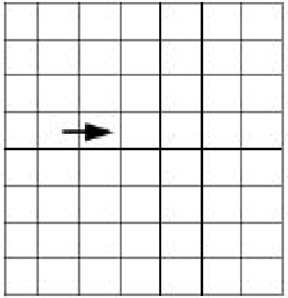
- Keep track of where agent has been
- More efficient updates





$$V_{i+1}(s) \leftarrow V_i(s) + \alpha(\overbrace{r + \gamma V_i(s')}^{learnt\ value} - V_i(s))$$

TD(0)



(T,R,S,A)

TD(0) Learning:

- Initialise V for all **s**
- For each trajectory/episode:
  - for all s
    - $e(s) = 0$
  - for each experience tuple (***s,r,s'***) under policy $\boldsymbol{\pi}$ in episode:
    - $e(s) = e(s) + 1$
    - $\delta = r + \gamma V(s') - V(s)$
    - for all ***s*** *in* ***S***
      - $V(s) \leftarrow V(s) + \alpha\delta e(s)$
    - $e(s) = 0$

We are back to normal TD Learning.

TD rollouts

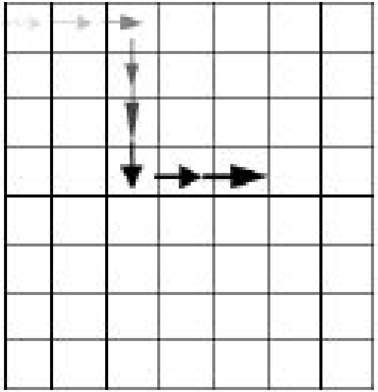
(s, a, r, s') (s', a', r', s'') (s'', a'', r'', s''')

(T,R,S,A)

1-step TD
and TD(0)

2-step TD

3-step TD



$V_{t+1}(s) \leftarrow V_t(s) +$ $\overbrace{\gamma V_t(s') - V_t}^{t \ value}$

$V(s) \leftarrow V(s) + \alpha$ $r_{t+1} + \gamma^2 V(s'') - V(s))$

$$V(s) \leftarrow V(s) + \alpha(r + \gamma r' + \gamma^2 r'' + \gamma^3 V(s''') - V(s))$$

TD(1)

TD(1) Learning: *(T,R,S,A)*

- Initialise *V* for all **s**
- For each trajectory/episode:
  - for all s
    - *e(s) = 0*
  - for each experience tuple (***s,r,s'***) under policy ***π*** in episode:
    - *e(s) = e(s) + 1* Mark whole trajectory
    - $\delta = r + \gamma V(s') - V(s)$
    - for all ***s*** *in* ***S***
      - $V(s) \leftarrow V(s) + \alpha\delta e(s)$
    - *e(s) = γe(s)* Decay trace

$$\boxed{V(s) \leftarrow V(s) + \alpha(r_t + \gamma r_{t+1} + \gamma^2 V(s'') - V(s))}$$

Tuning the decay

TD(0)

No traces

TD(1)

Traces
decay with γ

TD(λ)

Control the
decay rate

TD(λ)

TD(λ) Learning: ***(T,R,S,A)***

- Initialise V for all **s**
- For each trajectory/episode:
  - for all s
    - $e(s) = 0$
  - for each experience tuple (***s,r,s'***) under policy π in episode:
    - $e(s) = e(s) + 1$
    - $\delta = r + \gamma V(s') - V(s)$
    - for all ***s in S***
      - $V(s) \leftarrow V(s) + \alpha\delta e(s)$
    - $e(s) = \gamma\lambda e(s)$

Control the speed of decay

Intermission

15 minutes

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Onwards from TD

Recap: we can now learn by estimating **V** from experience

But:

- Not using actions ***A***

- We would rather learn ***Q***,

  for easier policy extraction!

  V requires a one-step lookahead model

SARSA

Learn from **s, a, r, s', a'**

(*T*,*R*,*S*,*A*)

- Initialise ***Q*** for all **s, a**
- For each episode
  - Initialise s_0
  - Choose a_0 in s_0 from ***Q***
  - For each step *t* in episode
    - Take a_t, observe r, s_{t+1}
    - Choose a_{t+1} in s_{t+1} from ***Q***

act

look ahead

learn

$$Q(s_t, a_t) \leftarrow \underbrace{Q(s_t, a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \cdot \left(\overbrace{\underbrace{r_t}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \cdot \underbrace{Q(s_{t+1}, a_{t+1})}_{\text{estimate of future value}}}^{\text{learned value}} - \underbrace{Q(s_t, a_t)}_{\text{old value}} \right)$$

SARSA

(*T*,*R*,*S*,*A*)

Where did we get the a_{t+1} ?

- Taking the next action under **Q**
- This is an **on policy** algorithm

What about **off policy**?

- Learn about optimal policy while exploring
- Reuse experience from other policies
- Learn from observations

Q-Learning

(*T*,*R*,*S*,*A*)

- Initialise ***Q*** for all ***s, a***
- For each episode
  - Initialise s_0
  - For each step ***t*** in episode
    - Choose a_t in s_t from **Q**
    - Take a_t , observe r, s_{t+1}

act

learn

$$Q(s_t,a_t) \leftarrow \underbrace{Q(s_t,a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \cdot \left(\overbrace{\underbrace{r_t}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \cdot \underbrace{\boxed{\max_a Q(s_{t+1},a)}}_{\text{estimate of optimal future value}}}^{\text{learned value}} - \underbrace{Q(s_t,a_t)}_{\text{old value}} \right)$$

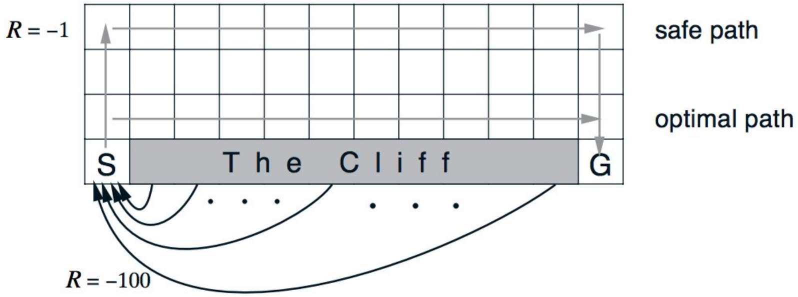
take best next action
(so far)

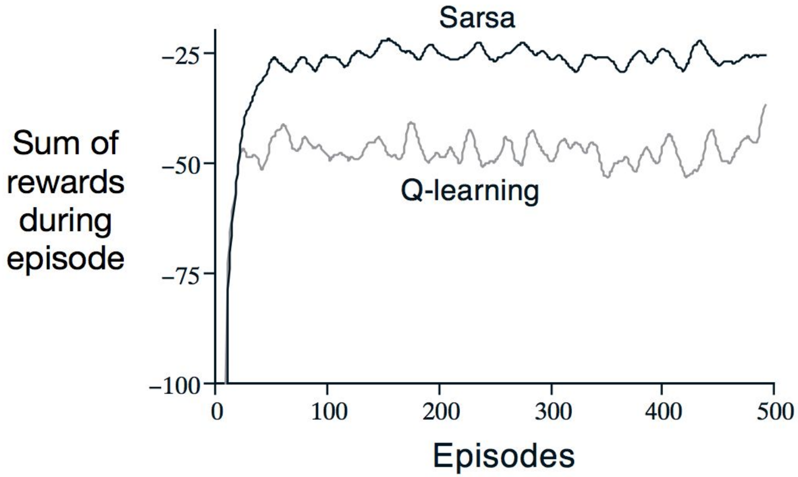
Q-Learning demo

(T,R,S,A)

Shreyas Skandan: https://www.youtube.com/watch?v=RTu7G0y4Os4

Typical Learning Curves





Generalising...

What about extending behaviour to different tasks?

What about building a simulator?

Ask questions about the domain

Solution: we need a **model**!!!

Model Based Methods

From Values to Environment Models

Model based reinforcement learning

Learn a model (***T*** and ***R***) from experience

- Supervised learning problem

Models let you predict next state and reward

- Reason about uncertainty

Algorithm setup

Model Based RL:

(T,R,S,A)

- No Transition Model
- No Reward Model
- Access to environment for experiment or access to training data ***(s,a,r,s')***
- **Goal**: Learn Transition and Reward Models
  - Policy through learned models.

Model Based RL

Learn a Transition and Reward Model

(T,R,S,A)

On receiving experience (s_t, a_t, r_t, s_{t+1}):

$$R(s_t, a_t) \Leftarrow R(s_t, a_t) + \alpha(r - R(s_t, a_t))$$

$$T(s_t, a_t, s_{t+1}) \Leftarrow T(s_t, a_t, s_{t+1}) + \alpha(1 - T(s_t, a_t, s_{t+1}))$$

$$T(s_t, a_t, \hat{s}) \Leftarrow T(s_t, a_t, \hat{s}) + \alpha(0 - T(s_t, a_t, \hat{s}))$$

$$Q(s,a) = R(s,a,s') + \gamma \sum T(s,a,s')V(s')$$

Dyna Q Algorithm

For each step ***t*** in episode

- Choose a_t in s_t from ***Q***
- Take a_t, observe r, s_{t+1}
- Update ***Q***: $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha\{r_t + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)\}$

Q-learning

- Given (s_t, a_t, r_t, s_{t+1})
- Update ***T*** and ***R***

model update

- Repeat *n* times:
  - Sample previously observed ***s***
  - Sample previously taken ***a*** (in ***s***)
  - Get ***r*** and ***s'*** from model
  - Update ***Q:*** $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha\{r_t + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)\}$

sample model to update Q

What else can I do with a model?

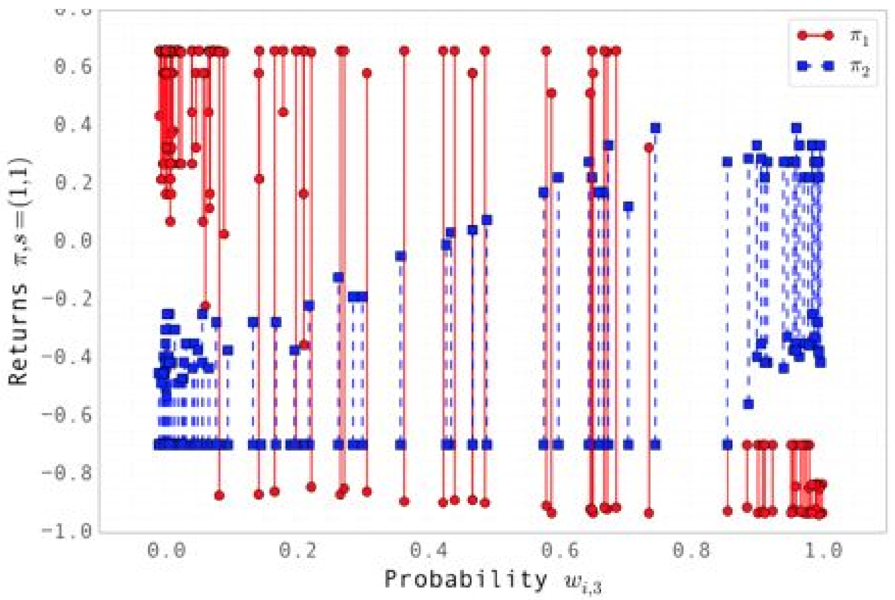
Quantify uncertainty in value functions

Uncertainty from:

- Data sparsity
- Inherent stochasticity
- Latent structure

Approaches:

- Monte Carlo sampling
- Simulation



A little bit of overkill?

Ok, so we've gone to all this trouble to learn $T, R \rightarrow Q$ …

Can't we just learn the policy?

Policy Search

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Algorithm setup

Direct Policy Learning:

(T,R,S,A)

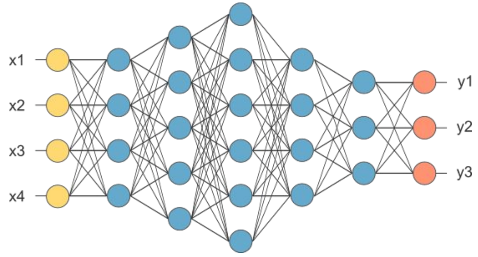
- No Transition Model
- No Reward Model
- Access to environment for experiment or access to training data ***(s,a,r,s')***
- **Goal**: Learn policy directly

Policy Gradient

Parametrise policy: $\pi_\theta(s,a) = P(a|s,\theta)$

Choices:

- Linear combination of basis functions
- Set of state features
- Deep neural network



Goal: find best θ

Optimisation problem!

Optimising the policy

Define cost function $J(\theta)$:

Start value, average reward per time step…

Find θ that maximises $J(\theta)$

e.g. gradient ascent on:

$$\Delta\theta = \alpha \nabla_\theta J(\theta)$$

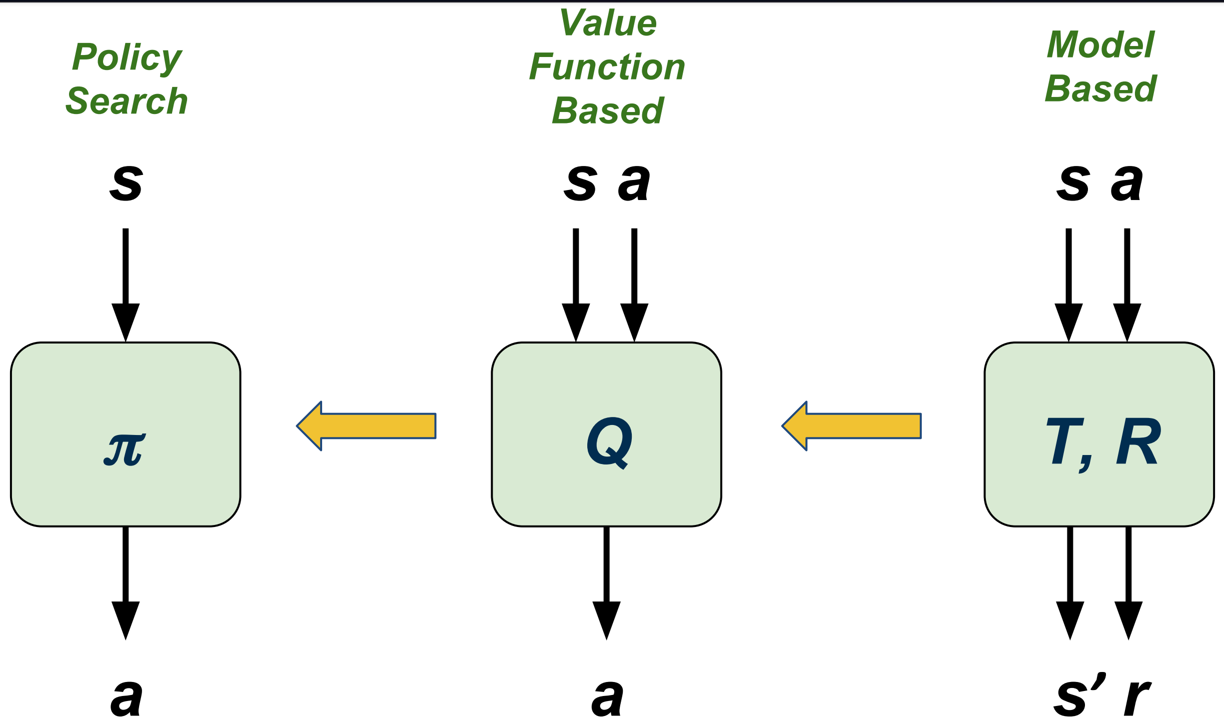
policy gradient

Why policy gradient?

+ High-dimensional action spaces
+ Continuous action spaces
+ Many recent successes in robotics

- Local convergence
- Policy evaluation high variance

Recap - RL Approaches



Inverse Reinforcement Learning

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Inferring a Reward Function

Designing reward functions is hard!

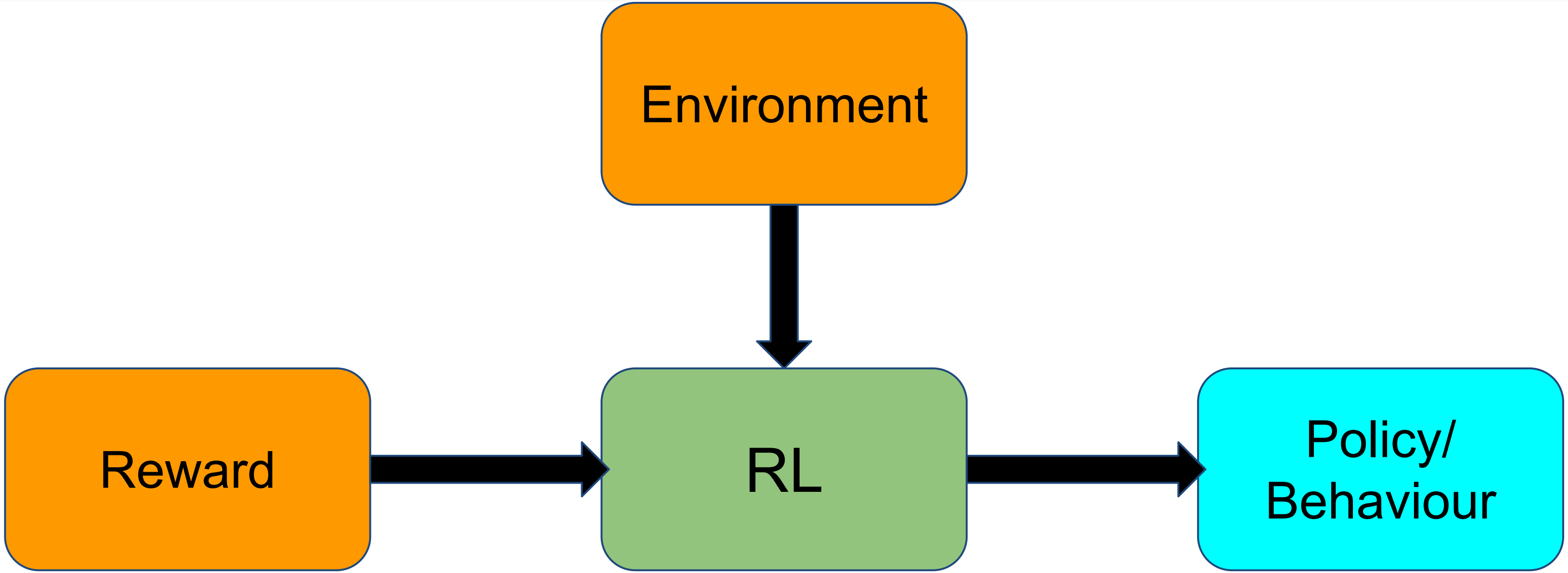
- Often not clear what should be done or how it should be rewarded
- Where do these come from?

Learn the incentives that explain observed behaviour

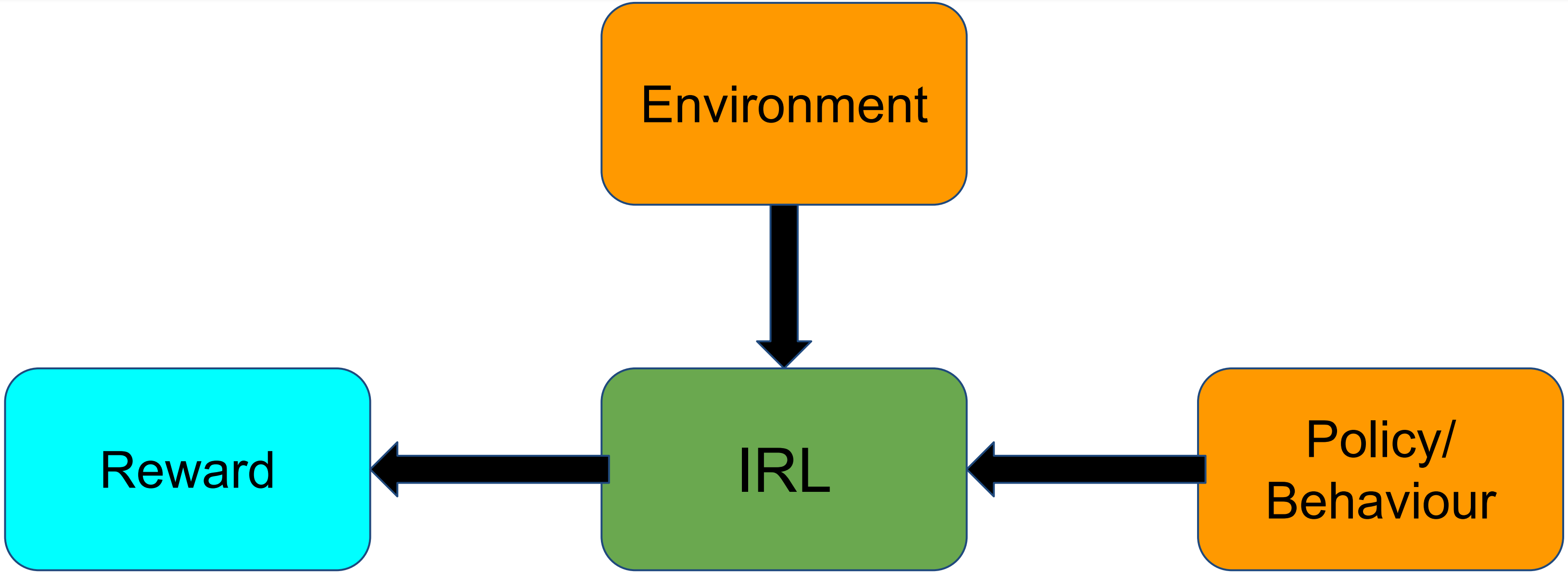
- From an “expert”

We do not observe the reward, but want to learn it

Inverse Reinforcement Learning



Inverse Reinforcement Learning



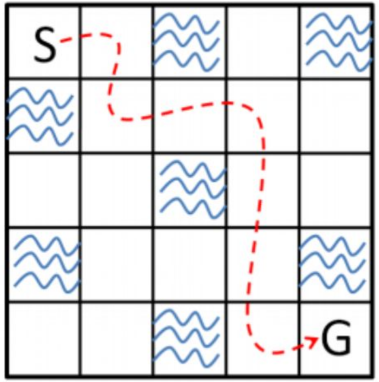
Algorithm setup

Inverse RL:

(T,R,S,A)

- Transition Model **(Can be learned)**
- No Reward Model
- Observe training data ***(s,a,s')***
- **Goal**: Learn a reward model to explain the behaviour observed through the training data

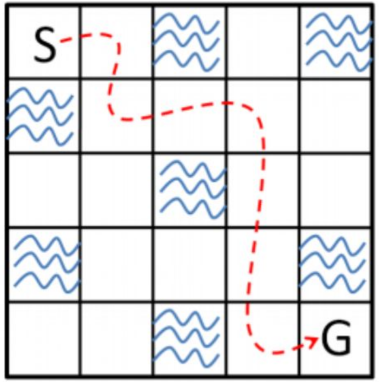
IRL: From paths to rewards



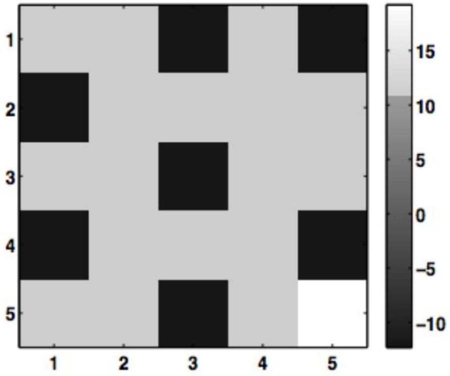
- Observe trajectory/trajectories (s,a,s')
- Would like to know:
  - What was the goal of the agent?
  - What was the reward?

Get to ***G*** and avoid water?

Maximum Likelihood IRL



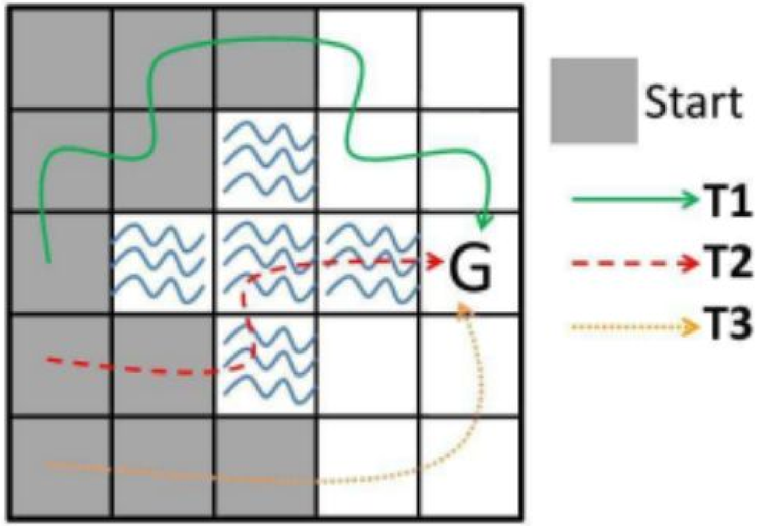
Possible reward function



ML IRL Algorithm (Intuition):

- Given sample trajectories D
- Initialise a reward function $\mathcal{R}$
- Calculate policy from $\mathcal{R}$, T
- Calculate $P(\boldsymbol{D}|\boldsymbol{\pi})$
- Calculate gradient, update $\mathcal{R}$

IRL: From paths to rewards



What about different teachers?

Information not in the data when we get it.

MLIRL with multiple intentions!!!

M Babes et. al. Apprenticeship learning about multiple intentions

IRL

Learn from demonstration

- Crowdsourcing
- Showing tasks to robots
- Learning from experts

(Some) Reinforcement Learning Applications

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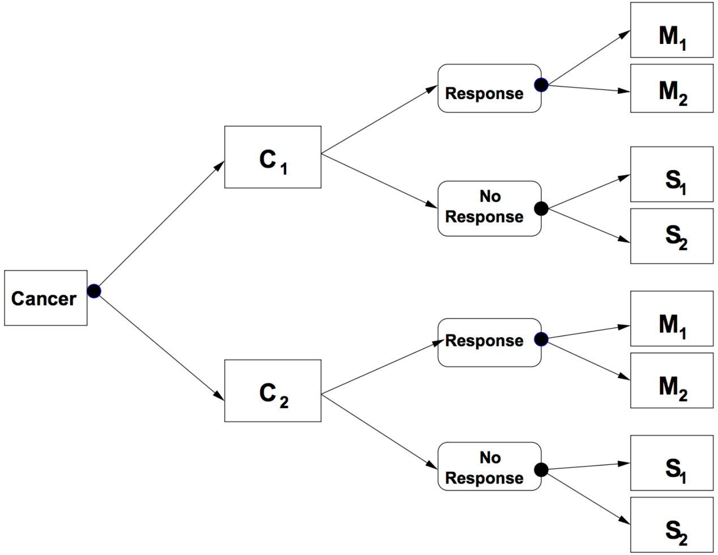
Application Areas

Randomised Controlled Trials



Efficacy in Sequential Multiple Assignment Randomized Trial

An Introduction to Dynamic Treatment Regimes: Marie Davidian

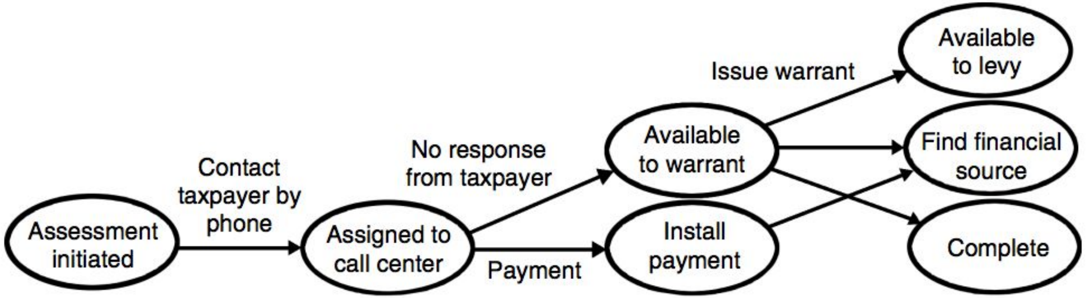
Application Areas

Advertising :(

Nuff Said!!!

Application Areas

Strategies to Improve Donations or Collecting Taxes :)



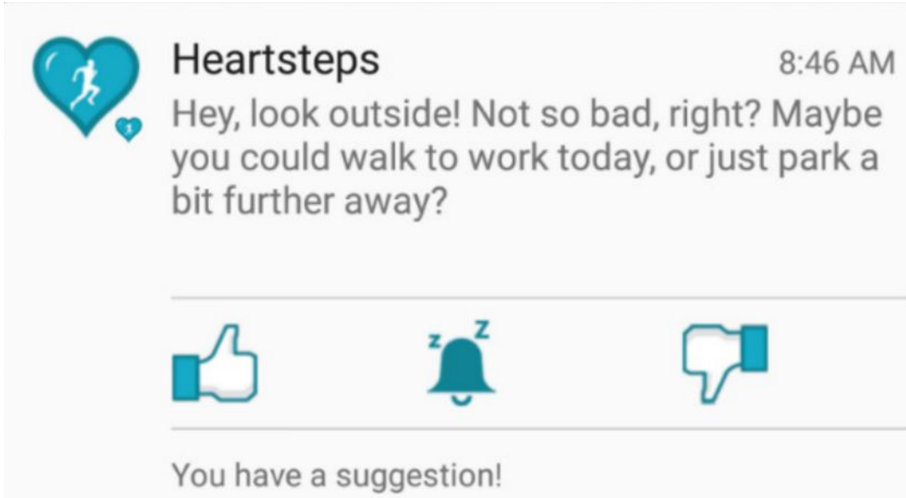
Action
Collections actions
Contact taxpayer by mail
Contact taxpayer by telephone
Create warrant
Create income execution
Create levy
Movement actions
Move to district office
Move to high-value team
Move to collection vendors
Move to indiv. case enforcement
Organization-specific action
Perform field visit
No action
Take no action

Tax Collections Optimization for New York State - Gerard Miller et. al.

Application Areas

Mobile Health Interventions

Heartsteps



Experimental Design & Machine Learning Opportunities in Mobile Health: Susan Murphy

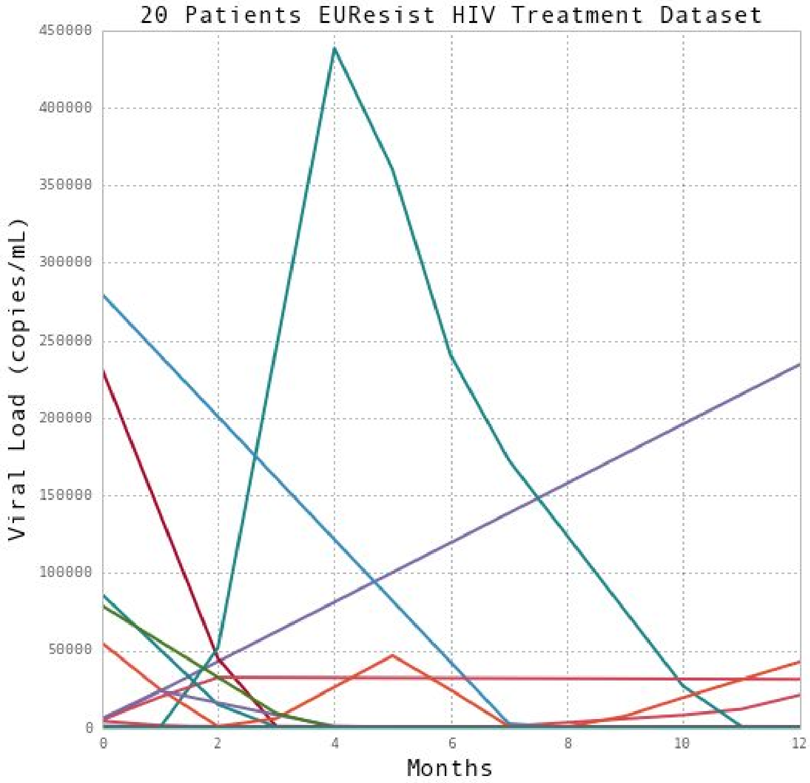
HIV Treatment: Possible Formulation

Features:
- baseline viral load, CD4 count,
- baseline CD4 percentage,
- Age, # previous treatments.

States:
- Viral Load tracked monthly over 24 months.
- Patient's treatment stage
- bins for the viral load, in copies/mL, were [0.0,50,100,1K,100K].

Actions:
- Therapy/drug cocktail groups occurring in the data set.

Reward:
- Negated AUC



V Marivate: Improved empirical methods in reinforcement-learning evaluation

Application Areas

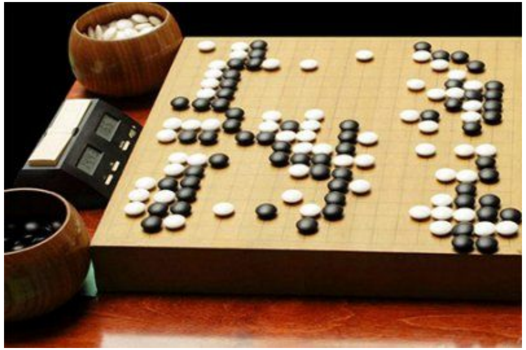
Robotics:

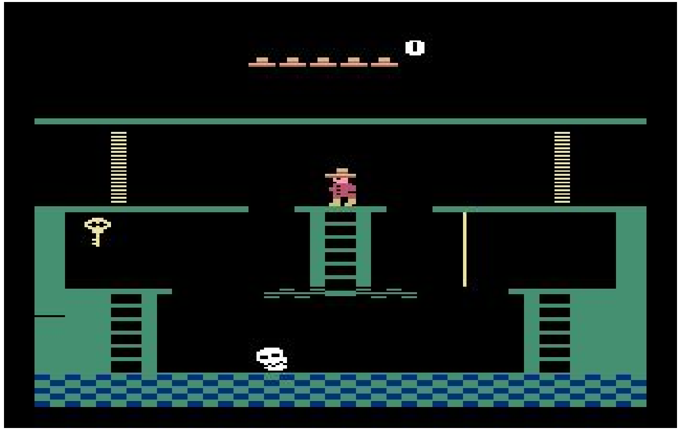
learning behaviours

RL Application Areas

Games

- Standardised testbeds
- Long decision horizons





Application Areas

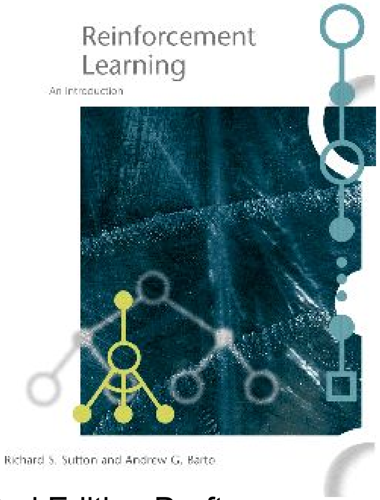
Automated Trading

1:

2: ???

3:

Thank you + Resources



2nd Edition Draft Recommended. Draft available online http://incompleteideas.net/sutton/book/the-book-2nd.html

UDACITY

RL class: https://www.udacity.com/course/reinforcement-learning--ud600

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